40.

Polynomials

Level: Introductory Grade 9

Time: 1–2 class periods (40–80 minutes)

Objectives:

1. To identify characteristics of polynomials such as terms, coefficients, variables, and degrees.
2. To distinguish between polynomials of different degrees.
3. To distinguish among polynomials of the same degree.
4. To be able to give geometric analogs for parts of polynomials.

Prerequisite Skills: Knowledge of the concepts of linear, area, and volume measurement

Materials: Poster board (white, blue, orange, red, yellow, green, and black)

Preparation: The materials listed below are required for 1 set of polynomial models. All dimensions should be doubled if the set is intended for use as a teacher-demonstrator model. The dimensions can be any value as long as they are not integral multiples of 2 cm.

–25 2 cm × 2 cm squares (black—representing numbers)

10 each of the following:
–2 cm × 11.4 cm strips (red—representing x)
–2 cm × 14.4 cm strips (yellow—representing y)
–2 cm × 19 cm strips (green—representing z)

5 each of the following:
–11.4 cm × 11.4 cm squares (red—representing x^2)
–14.4 cm × 14.4 cm squares (yellow—representing y^2)
–19 cm × 19 cm squares (green—representing z^2)
–11.4 cm × 14.4 cm rectangles (white—representing xy)
–11.4 cm × 19 cm rectangles (blue—representing xz)
–14.4 cm × 19 cm rectangles (orange—representing yz)

3 each of the following:
–11.4 cm × 11.4 cm × 11.4 cm cubes (red—representing x^3)
–14.4 cm × 14.4 cm × 14.4 cm cubes (yellow—representing y^3)
–19 cm × 19 cm × 19 cm cubes (green—representing z^3)

1 each of the following rectangular prisms:
–11.4 cm × 11.4 cm × 14.4 cm (red, white—representing x^2y)
–11.4 cm × 11.4 cm × 19 cm (red, blue—representing x^2z)
–11.4 cm × 14.4 cm × 14.4 cm (yellow, white—representing xy^2)
–11.4 cm × 19 cm × 19 cm (green, blue—representing xz^2)
–11.4 cm × 14.4 cm × 19 cm (white, blue, orange—representing xyz)
–14.4 cm × 14.4 cm × 19 cm (yellow, orange—representing y^2z)
–14.4 cm × 19 cm × 19 cm (green, orange—representing yz^2)

NOTE: For the cubes and rectangular prisms, the squares and rectangles manufactured earlier can be duplicated, then taped together with invisible tape.

An interesting additional activity can be developed if the construction of the models is made into a problem-solving exercise.

Have the class try to determine the number of rectangles (2 dimensions) and the number of rectangular prisms (3 dimensions) that can be made by using just 3 basic measures.

Procedure:

1. Define the primary parts of the models for the class.

   (a) Black squares: 1 unit by 1 unit—value = 1.
   For example, 7 squares is said to be 7.

   (b) Red strip: 1 unit wide by x units long—value = x.
   For example, 8 red strips would be called 8x.

   (c) Yellow strip: 1 unit wide by y units long—value = y.
   For example, 3 yellow strips would be called 3y.

   (d) Green strip: 1 unit wide by z units long—value = z.
   For example, 12 green strips would be called 12z.

2. At this point, you may wish to have the class do questions 1 and 2 of the Exercises for Students.

3. Define terms and expressions using these models.

   (a) A term: one of the basic models.
   For example, 3 (3 black squares)
   5x (5 red strips)
   128z (128 green strips)

   (b) an expression: several basic models connected with plus signs.
   For example, 2x + 5 (2 red strips and 5 black squares)
   4x + 3y + 5z (4 red strips, 3 yellow strips, and 5 green strips)

4. At this point, you may wish to have the class do questions 3 and 4 of the Exercises for Students.

5. Expand the notion of terms and expressions to include the concept of 2 dimensions, that is, 2 variables. Locate all of the large squares and rectangles. Identify the lengths of the sides by matching strips.

   For example, students will discover that the white rectangle can be matched with a red strip on 1 side and a yellow strip on the other. Calling on the students' knowledge of area, point out that the area of the white rectangle can be found by using $A = ab$ (altitude: red—x; base: yellow—y). Hence, the area of the white rectangle is xy. Similarly, the other squares and rectangles can be labeled according to their areas. Have the class identify all such areas.

   (a) red: red × red—x^2
   (b) yellow: yellow × yellow—y^2
   (c) green: green × green—z^2
   (d) blue: red × green—xz
   (e) orange: yellow × green—yz
   (f) white: red × yellow—xy

NOTE: Some students will find it helpful to record all their discoveries on a chart, which they can build as they go along.

6. Expand the notion of expressions to include the newly discovered 2-dimensional objects.
   For example, 2 orange rectangles—label 2yz.
   1 red square, 3 blue rectangles, 4 red strips, 5 black squares—label $x^2 + 3xz + 4x + 5$.

7. At this point, you may wish to have the class do questions 5 and 6 of the Exercises for Students.

8. Expand the notion of expressions to include the concept of 3 dimensions, that is, 3 variables. Locate all 3-dimensional objects in the set. Separate the cubes from the prisms. Recall that the value of such solids is calculated by multiplying length × width × height. Identify the various dimensions by matching strips, as was done with the rectangles.

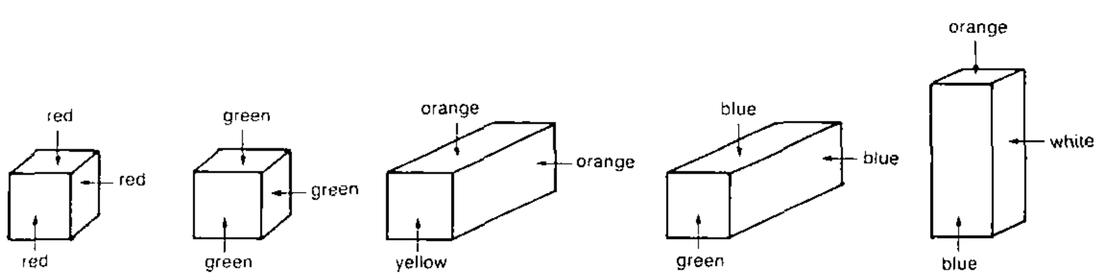
   For example, red cube length: red—x
   width: red—x
   height: red—x

   Hence, the volume of this cube is x^3. Similarly, all of the cubes and prisms can be labeled according to their volume. Have the class identify all such volumes.

   CUBES:
   (a) red cube: red × red × red—x^3
   (b) yellow cube: yellow × yellow × yellow—y^3
   (c) green cube: green × green × green—z^3

   PRISMS:
   (a) 2 red squares and 4 blue rectangles: red × red × green—x^2z
   (b) 2 red squares and 4 white rectangles: red × red × yellow—x^2y
   (c) 2 yellow squares and 4 white rectangles: yellow × yellow × red—y^2x (or xy^2)
   (d) 2 yellow squares and 4 orange rectangles: yellow × yellow × green—y^2z
   (e) 2 green squares and 4 orange rectangles: green × green × yellow—z^2y (or yz^2)
   (f) 2 green squares and 4 blue rectangles: green × green × red—z^2x (or xz^2)
   (g) 2 white rectangles, 2 blue rectangles, and 2 orange rectangles: red × yellow × green—xyz

9. Have the class practise labeling some of the 3-dimensional figures.



This would be labeled $x^3 + z^3 + y^2z + xz^2 + xyz$.

10. At this point, you may wish to have the class do questions 7 and 8 of the Exercises for Students.

Exercises for Students

1. Write the label (name) for each of the following:

(a) 4 black squares
(b) 7 red strips
(c) 4 green strips
(d) 6 yellow strips
(e) 50 black squares
(f) 1000 yellow strips
(g) 91 red strips
(h) 14 green strips

2. Show how to demonstrate the following by using the polynomial model set:

(a) 6
(b) x
(c) $3y$
(d) $7z$
(e) 72
(f) $31y$
(g) $1000x$
(h) $5z$

3. Show how to demonstrate the following:

(a) $3x + 5$
(b) $2z + 4y$
(c) $4y + 2z$
(d) $3z + 1$
(e) $x + y + z$
(f) $2x + 2y + 2z$

Are b and c the same?

4. Write the polynomials that are described by:

(a) 6 black squares
(b) 3 black squares and 2 red strips
(c) 5 green strips and 4 yellow strips
(d) 7 yellow strips, 2 green strips, and 1 red strip
(e) 6 red strips, 3 yellow strips, and 9 black squares

5. Show how to demonstrate the following polynomials:

(a) $2x^2 + 3y^2$
(b) $2x^2 + 5x^2 + 6z^2$
(c) $3xy + 4y^2 + 2x + 8$
(d) $12x^2 + 3yz + 5y^2 + 4xz + 3y + 31$

6. Write the polynomials that are described by:

(a) 3 red squares and 1 black square
(b) 2 yellow squares and 6 green strips
(c) 5 green squares, 3 blue rectangles, and 4 black squares
(d) 2 red squares, 9 orange rectangles, and 5 yellow strips
(e) 1 yellow square, 2 blue rectangles, 3 yellow rectangles, 4 green strips, and 5 black squares

7. *Work with a partner. Select a set of geometric objects.* Have your partner write the polynomial.

Now, have your partner select the objects. You write the polynomial. Repeat this 4 times.

8. Find the objects described by the following polynomials:

(a) y^3
(b) xyz
(c) x^2z
(d) $2x^3 + 3xy$
(e) $x^2y + 2xy^2 + 5$
(f) $x^3 + 3x^2z + 4xy + 2z + 3$

Demonstrating the Degree of a Polynomial

1. When a polynomial *term* describes a 3-dimensional object, that term is said to be of *degree 3*. The following terms would all be of degree 3:

   (a) x^3 (red cube)
   (b) $2x^2y$ (prisms with red squares and white rectangles)
   (c) $4zy^2$ (prisms with yellow squares and orange rectangles)
   (d) $9xy^2$ (prisms with white, blue, and orange rectangles)

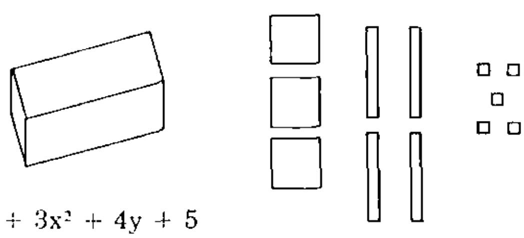
   If a term describes a 2-dimensional object, that term is said to be of degree 2. The following terms would all be of degree 2:

   (a) x^2 (red square)
   (b) $3xy$ (white rectangles)
   (c) $9z^2$ (green squares)

   If a term describes a 1-dimensional object, that term is said to be of degree 1. The following terms would all be of degree 1:

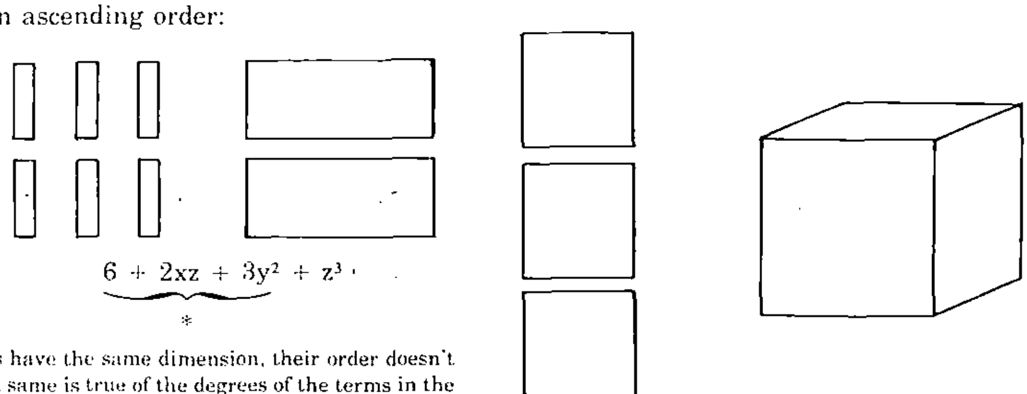
   (a) $3z$ (green strips)
   (b) x (red strip)
   (c) $9y$ (yellow strips)

2. When a polynomial is arranged so that the term with the largest degree is written first, then the next largest and so on down, it is written in *descending order*. The following is an example of a polynomial written in descending order:



$x^2z + 3x^2 + 4y + 5$

3. When a polynomial is arranged so that the term with the smallest degree is written first, then the next larger and so on up, it is written in *ascending order*. The following is an example of a polynomial written in ascending order:



$6 + \underbrace{2xz + 3y^2}_{*} + z^3$

*If 2 objects have the same dimension, their order doesn't matter. The same is true of the degrees of the terms in the related polynomial.

4. The *coefficient* of a polynomial term tells how many objects there are. For example, $2y^3$ would be described by 2 yellow cubes. Because there are 2 cubes, the coefficient of $2y^3$ is 2.

Additional Exercises

It is recommended that the students work in pairs or groups of 3 to do the following exercises. One set of 3-dimensional models is required for each group.

1. Find the object set described by the term, then give its degree.

(a) $3z$	(d) $2xyx$	(g) $4y^3$
(b) $2x^2$	(e) $3yz$	(h) $4z^2$
(c) $4xy$	(f) $2z$	(i) $3y$

2. Give the degree of each *polynomial*. Check by picking out the object set.

(a) $15 + 7x + 2xy$	(e) $7y + 2z$
(b) $3xz + x^3$	(f) x^3
(c) $y^3 + y^2 + y + 1$	(g) $y^2 + x^2$
(d) $3 + 3z + 3z^2 + 3z^3$	(h) $3xyz + 2xy + 8x + 10$

3. Which of the following polynomial terms are of *degree 3* or describe 3-dimensional objects?

(a) x^3	(d) zy^3	(g) $7yx^2$
(b) x^2y	(e) $2y^3$	(h) 3
(c) $3x$	(f) $3x^2$	(i) xyz

4. Write the following polynomials in *descending* order. Check by ordering the related object set.

   (a) $7x + 3xz + 9 + y^3$
   (b) $3xyz + 8 + 5z + 3xy$
   (c) $1 + 2x + x^2$

5. Write the following polynomials in *ascending* order. Check by ordering the related object set.

   (a) $2x^3 + 3xy + x + 6$
   (b) $4x + 2y^3 + x^2z + 3$
   (c) $xz^2 + 2z^3 + 3yz$

6. Write the coefficient of each of the following terms.

(a) $2x^3$	(f) $7z$	(k) $5xy$
(b) $3y^3$	(g) $20x^3$	(l) $2xyz$
(c) $4x^2$	(h) $3y^2$	(m) $4yz$
(d) $0y^2$	(i) $2y^3$	(n) $2xy^2$
(e) $8z$	(j) $1001x$	(o) xz^2

7. Are $3y^2$ and $2y^3$ the same term? Find the 2 object sets.

   (a) Are they the same?
   (b) What does 3 mean in $3y^2$?
   (c) What does 3 mean in $2y^3$?

8. Pick out the objects needed to describe this polynomial: $2x^3 + 3y^2 + 2xy + z + 6$.

   (a) What is the degree of each term?
   (b) What is the degree of the polynomial?
   (c) Write the polynomial in ascending order.
   (d) Write the polynomial in descending order.
   (e) What is the coefficient of each term?