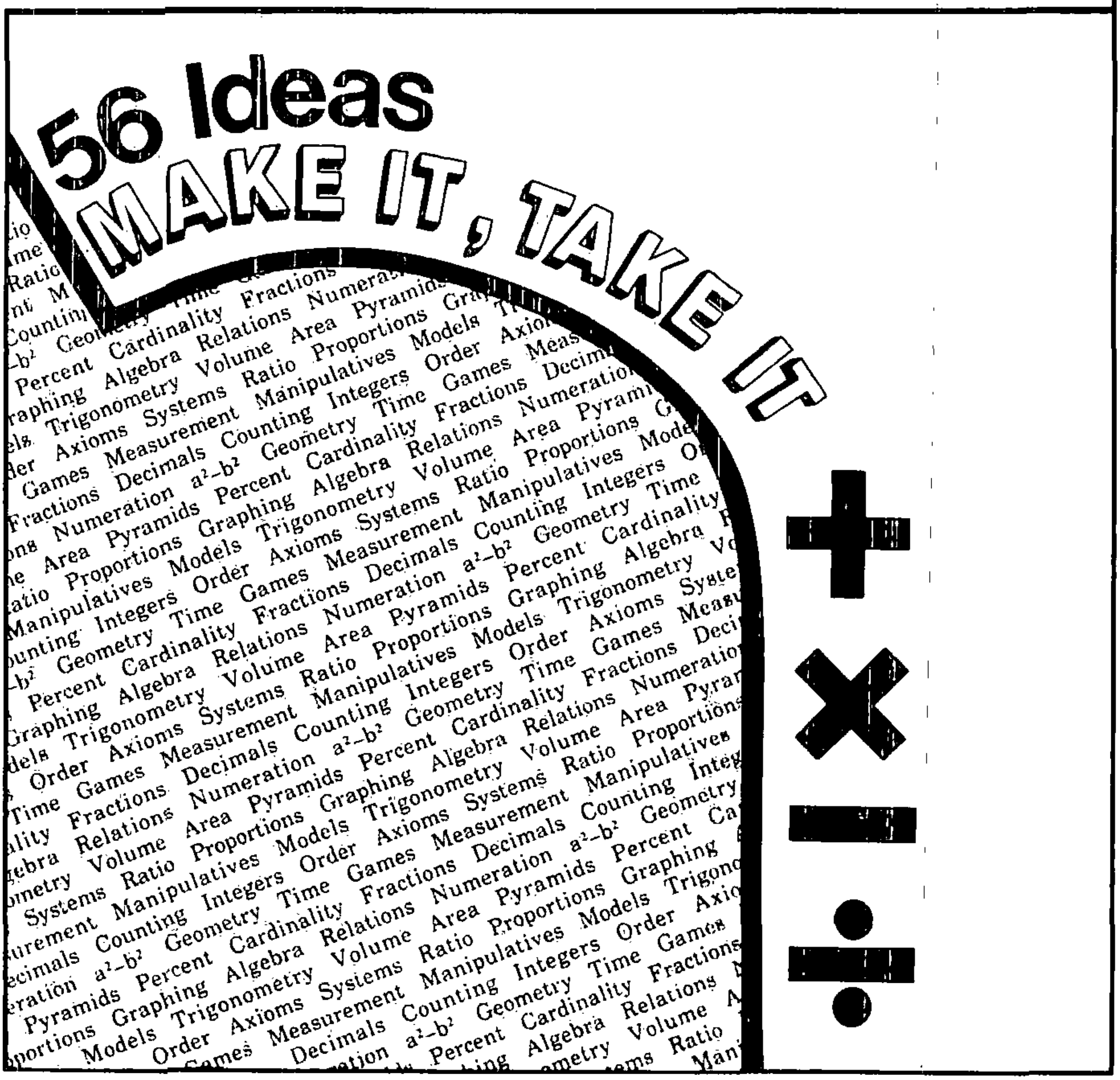
MATH Monograph No. 9

Coeditors: William Bober and John Percevault

December 1987

56 Ideas
MAKE IT, TAKE IT



MCATA Publication of the Mathematics Council of The Alberta Teachers' Association

MATH
Monograph No. 9

Make It—Take It

December 1987

Contents

Math Monograph No. 9 is published by the ATA for the Mathematics Council (MCATA). COEDITORS: William Bober, 10819-50 Street, Edmonton, Alberta T6A 2E3, and John Percevault, 2510-22 Avenue S, Lethbridge, Alberta T1K 1J5. EDITORIAL AND PRODUCTION SERVICES: Central Word Services staff, ATA. Address all correspondence regarding the contents of this publication to John Percevault. Opinions of writers are not necessarily those of the MCATA or the ATA. Members of the MCATA receive one copy of this publication free. Nonmembers wishing a copy may obtain one by forwarding $6 to The Alberta Teachers' Association, 11010-142 Street, Edmonton, Alberta T5N 2R1.

MATHEMATICS COUNCIL, ATA
Executive, 1987–88

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(c) any member of the Department of Education, or of a college or university in Alberta.

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Calgary School District No. 19
Edmonton Roman Catholic Separate School District No. 7
Edmonton School District No. 7
John Wilson Elementary School, Innisfail
(County of Red Deer School District No. 23)

The MCATA also acknowledges the contribution of William C. Bober, who has retired from the Edmonton Roman Catholic Separate School District. Bill organized the "Make and Take" sessions for the National Council of Teachers of Mathematics, Canadian Conference, held in Edmonton, October 1986. School systems and teachers were contacted, submissions were screened, and the sessions were organized. Conference materials were edited and arranged in the order presented in this publication.

The MCATA extends a special note of appreciation to the many volunteers who assisted Bill in preparing materials for the Canadian Conference.

John Percevault, editor of *delta-K,* compiled the materials provided by Bill. Many of the actual models that were provided by teachers have been omitted due to space limitations. However, instructions for development of the models are included.

Finally, the Mathematics Council acknowledges with thanks the work of Central Word Services staff at the provincial ATA office, most particularly the special efforts of Sharon Wood, Lil Fletcher, and Terry Ho, in the production of this monograph.

Robert Michie
President, MCATA

1.

2-D Figures and 3-D Objects

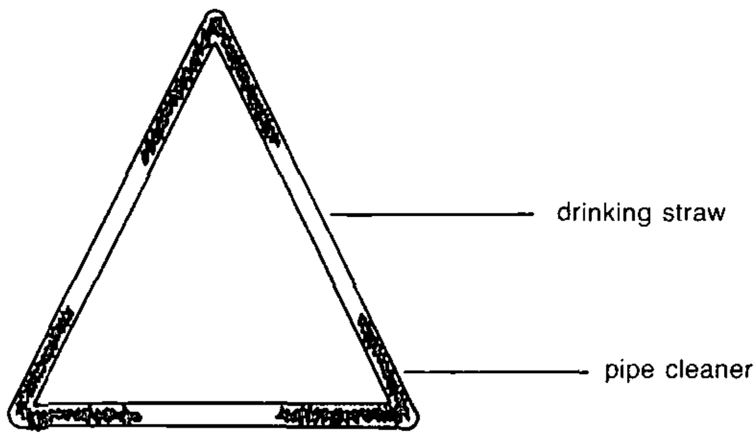
Topic: Geometry

Level: Grades 1–6

Materials: Pipe cleaners, drinking straws, scissors

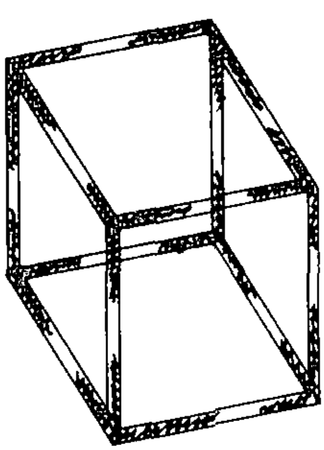
Procedure:

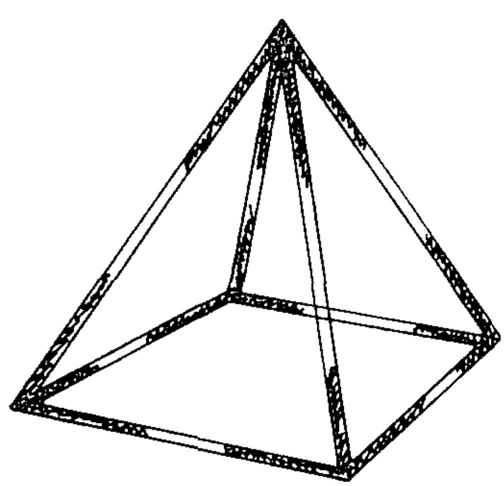
1. Make a triangle.



2. Make a cube.

3. Make a pyramid.





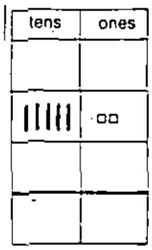
2.

Base 10 Blocks

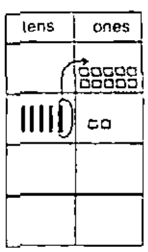
Topics:	Subtraction—	Division—	Dienes Blocks, Base 10—
	(a) using 1 set	(a) share evenly	(a) ones
	(b) using 2 sets	(b) how many for each	(b) tens
			(c) hundreds

Problem A.

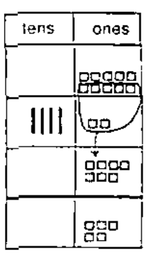
Jim had 52 marbles but lost 27. How many does he have left?

1. 

Show 52, putting ones and tens in the correct columns.

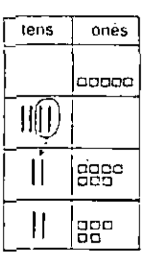
2. 

You can't take 7 from 2. Regroup a 10 into 10 ones. This gives 4 tens and 12 ones.

3. 

Take away 7 ones.

Show how many are left.

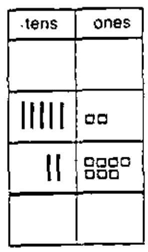
4. 

Take away 2 tens.

Show how many tens are left.

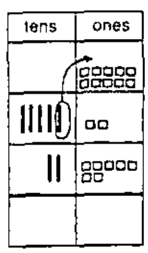
Problem B.

Jim has 52 marbles. John has 27 marbles. How many more marbles does Jim have?

1. 

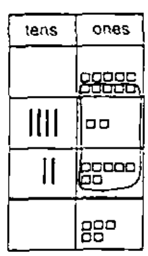
Show 52 and 27, putting ones and tens in the correct columns.

$$\begin{array}{r} 52 \\ -27 \\ \hline \end{array}$$

2. 

Compare the ones. Seven is greater than 2. Regroup a 10 into 10 ones. This gives 4 tens and 12 ones.

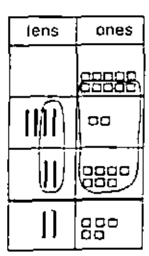
$$\begin{array}{r} \overset{4}{\not{5}}\,\overset{1}{}2 \\ -27 \\ \hline \end{array}$$

3. 

Match up the ones.

Show the difference.

$$\begin{array}{r} \overset{4}{\not{5}}\,\overset{1}{}2 \\ -27 \\ \hline 5 \end{array}$$

4. 

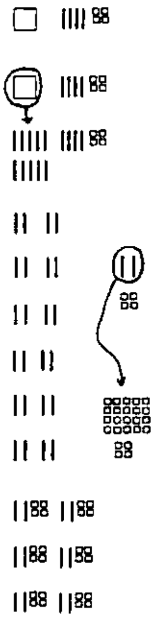
Match up the tens.

Show the difference.

$$\begin{array}{r} \overset{4}{\not{5}}\,\overset{1}{}2 \\ -27 \\ \hline 25 \end{array}$$

Problem C.

Susan has a package of 144 candy canes that she wants to share evenly among 6 friends. How many canes will each friend get?



Show the 144 canes to be divided evenly among 6 friends.

$$6\overline{)144}$$

You can't give 100 to each. One hundred is 10 tens, so you have 14 tens altogether.

$$6\overline{)14|4}$$

Give 2 tens to each friend, and record 2 in the tens column. This portioning takes 12 tens, leaving 2 tens.

$$\begin{array}{r} 2| \\ 6\overline{)14|4} \\ 12| \\ 2| \end{array}$$

Each 10 is 10 ones. With the 4 left, this makes 24 ones.

$$\begin{array}{r} 2| \\ 6\overline{)14|4} \\ 12| \\ 2|4 \end{array}$$

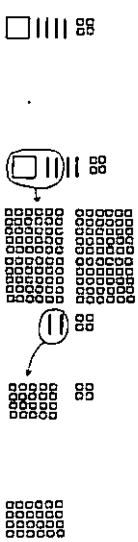
Give 4 canes to each friend, and record a 4 in the ones column. This sharing takes 24 canes, and no canes are left.

So, each friend got 24 canes.

$$\begin{array}{r} 2|4 \\ 6\overline{)14|4} \\ 12| \\ 2|4 \\ \underline{2|4} \\ 0 \end{array}$$

Problem D.

Susan has a package of 144 candy canes, enough for her to give 6 to each of her friends. How many friends can she share the candy canes with?



Show the 144 canes to be divided into groups of 6.

$$6\overline{)144}$$

You can't make 100 groups of 6.

You *can* make 20 groups of 6. Twenty is 2 tens.

Record 2 in the tens column.

Take 100 and 2 tens away.

Two tens are left.

$$\begin{array}{r} 2 \\ 6\overline{)144} \\ 12 \\ 2 \end{array}$$

Each 10 is 10 ones. With the 4 left, this makes 24 ones.

$$\begin{array}{r} 2 \\ 6\overline{)144} \\ 12 \\ 24 \end{array}$$

Make another 4 groups of 6. Record the 4 in the ones column. Take the 24 away.

$$\begin{array}{r} 24 \\ 6\overline{)144} \\ 12 \\ 24 \\ \underline{24} \end{array}$$

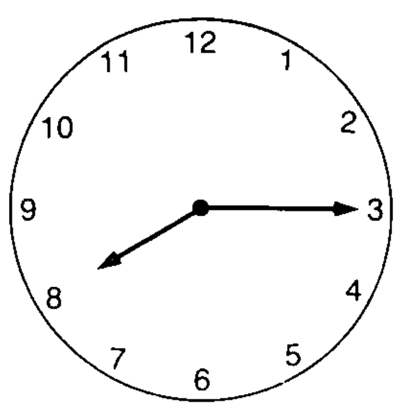
There are 24 groups of 6 altogether, so Susan can share with 24 friends.

3.

"I Have" Game

1. I have ______________ .
   Who has 10:25?
2. I have ______________ .
   Who has 2:05?
3. I have ______________ .
   Who has 6:30?
4. I have ______________ .
   Who has 12:50?
5. I have ______________ .
   Who has 9:05?
6. I have ______________ .
   Who has 11:45?
7. I have ______________ .
   Who has 12:10?
8. I have ______________ .
   Who has 4:45?
9. I have ______________ .
   Who has 9:05?
10. I have ______________ .
    Who has 6:50?
11. I have ______________ .
    Who has 11:05?
12. I have ______________ .
    Who has 3:25?
13. I have ______________ .
    Who has 7:20?
14. I have ______________ .
    Who has 10:10?
15. I have ______________ .
    Who has 3:55?
16. I have ______________ .
    Who has 1:35?
17. I have ______________ .
    Who has 6:10?
18. I have ______________ .
    Who has 8:25?
19. I have ______________ .
    Who has 2:45?
20. I have ______________ .
    Who has 12:40?
21. I have ______________ .
    Who has 7:35?
22. I have ______________ .
    Who has 11:50?
23. I have ______________ .
    Who has 1:05?
24. I have ______________ .
    Who has 8:15?

I have ______________ .

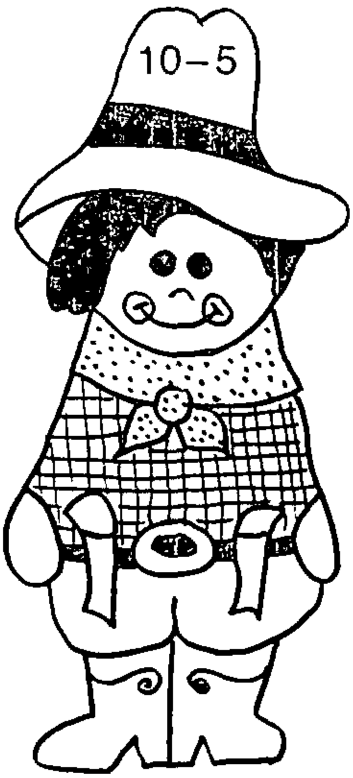


Who has 10:25?

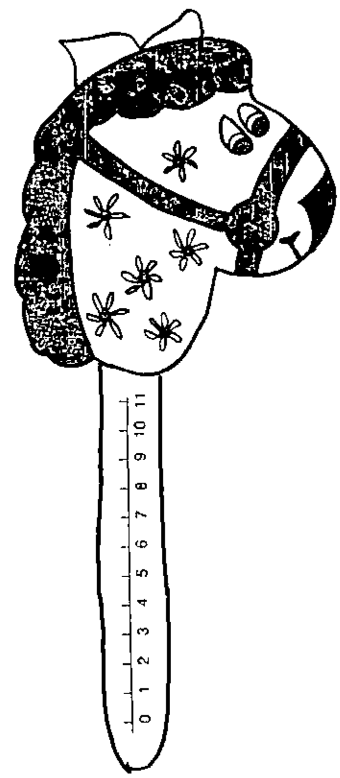
4.

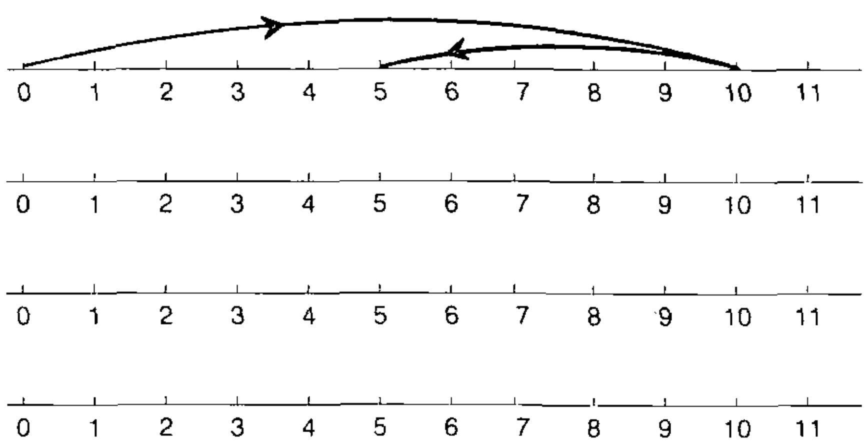
Sharp-Shootin' Charlie

Sharp-Shootin' Charlie can hit a target blindfolded, but he just can't seem to stay on his horse. He never learned how to subtract, and he needs your help. If you can use the number line to match the problem on the stick of Charlie's horse to the problem on his hat, you will make Charlie very happy!



7 – 4
11 – 5
9 – 3
10 – 4
11 – 3
8 – 2

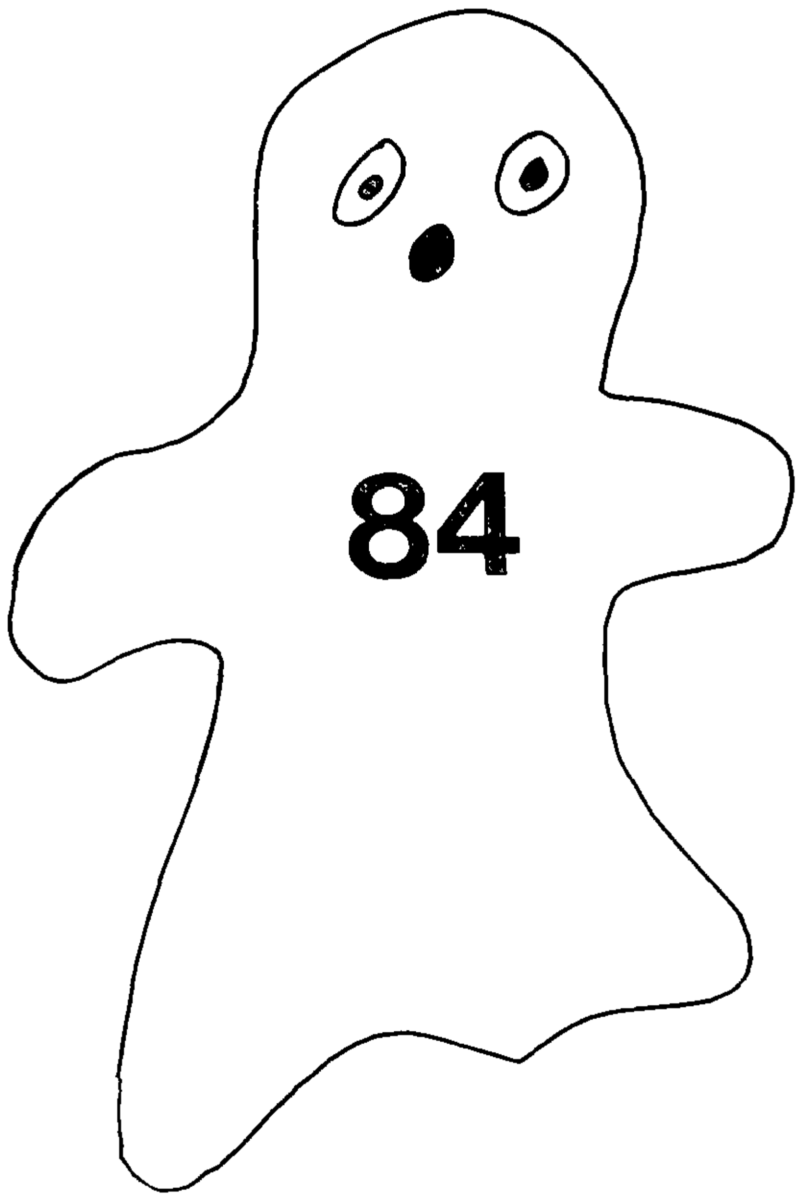




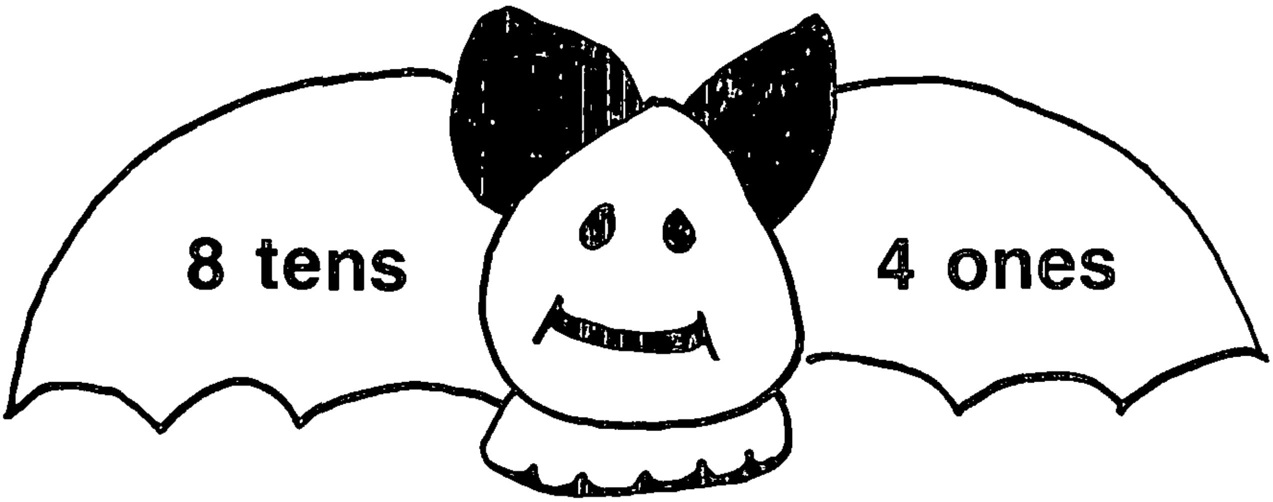
5.

Goober Ghost and Benny the Bat

Goober Ghost and Benny the Bat want to go out and play tricks on everyone. Benny has to know how many tens and ones are in each of Goober Ghost's numbers. Can you help? Match the correct ghost with each bat.



80	93	34
60	29	49
17	5	12
51	67	72
9	46	61



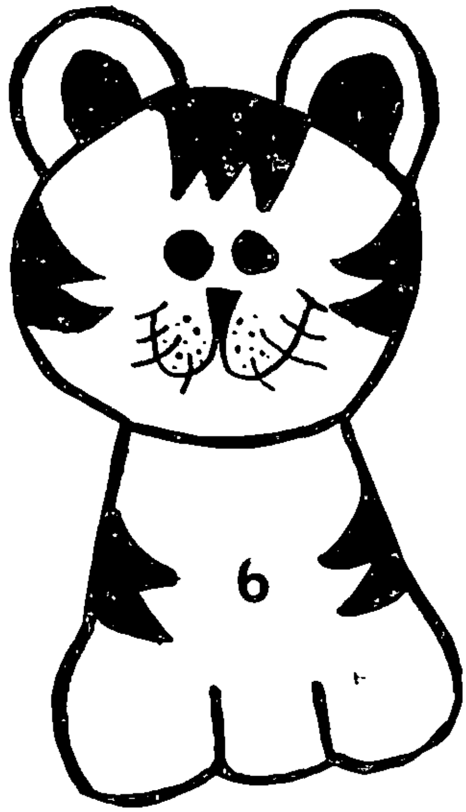
8 tens	0 ones	2 tens	9 ones
6 tens	0 ones	0 tens	5 ones
1 ten	7 ones	6 tens	7 ones
5 tens	1 one	4 tens	6 ones
0 tens	9 ones	3 tens	4 ones
9 tens	3 ones	4 tens	9 ones

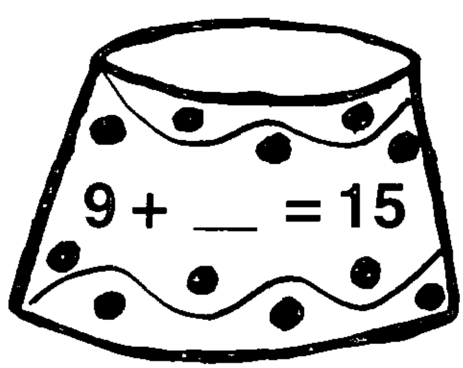
6.

Tigger the Tiger

Tigger the Tiger will get a treat if he sits on the correct seat. Will you help him? Solve the problem and match it with the correct tiger. Check your answers on the back of the seat.

8 + __ = 13	1
9 + __ = 13	9
7 + __ = 12	4
7 + __ = 14	0
9 + __ = 10	3
7 + __ = 11	5
8 + __ = 14	4
2 + __ = 11	7
8 + __ = 10	5
5 + __ = 10	4
8 + __ = 12	5
10 + __ = 10	2
7 + __ = 10	6
8 + __ = 16	8

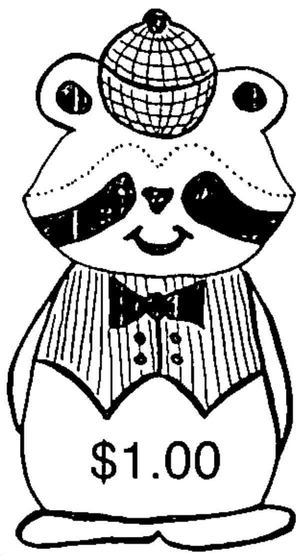




7.

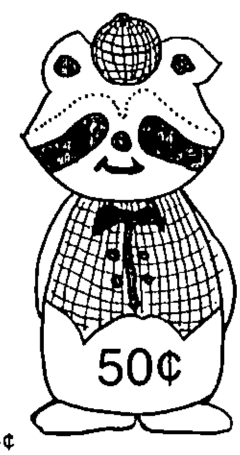
Ricky and Robby Raccoon

Ricky and Robby Raccoon have hidden their bags of gold. Now they would like to find them. Can you help? Robby will only take the bags that are worth $1.00. Ricky will only take the bags that are worth 50¢. Read the amounts on each bag of gold and match them to the correct raccoon. Check your answers.



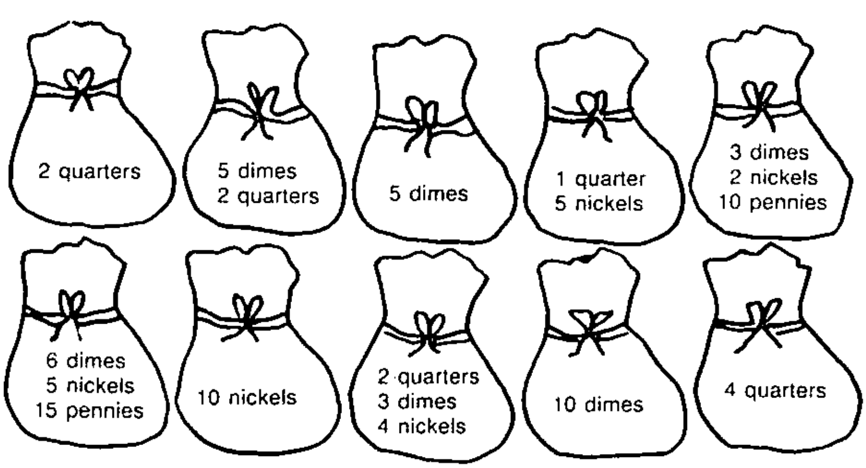
$1.00 ____________________

10 dimes
4 quarters
5 dimes, 2 quarters
6 dimes, 5 nickels, 15 pennies
2 quarters, 3 dimes, 4 nickels



50¢ ____________________

2 quarters
5 dimes
10 nickels
1 quarter, 5 nickels
3 dimes, 2 nickels, 10 pennies

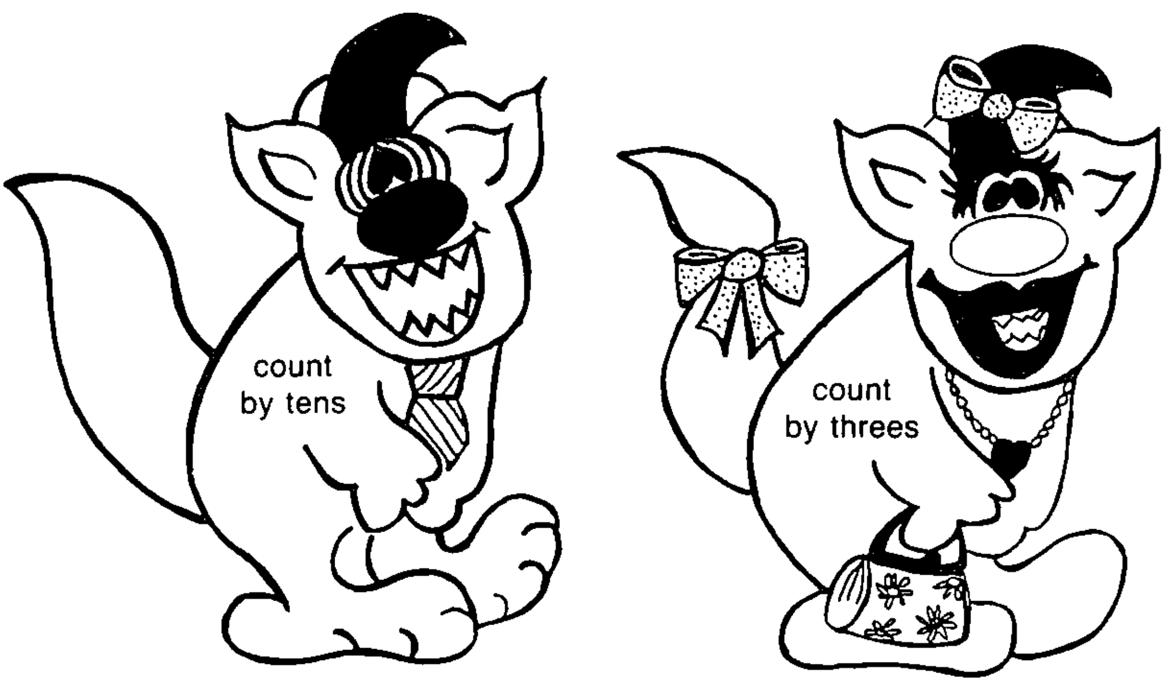


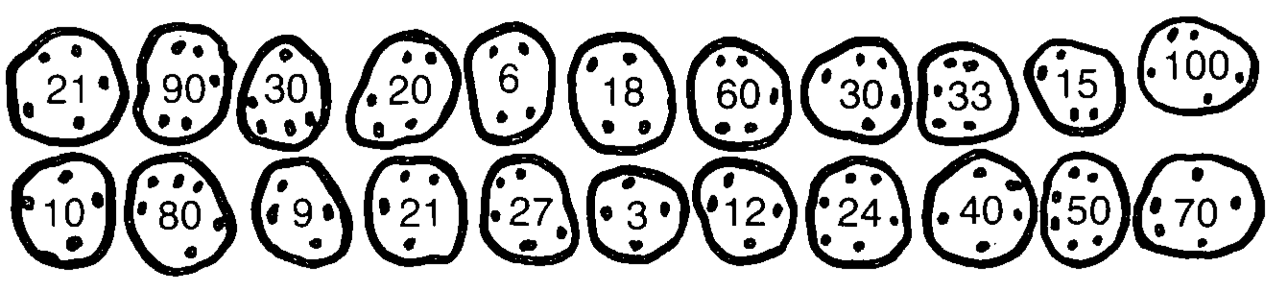
8.

The Cookie Monsters

Mr. and Mrs. Cookie Monster love each other very much, but there is one thing they love even more—cookies! Cookies are the one thing that they will not share with each other. If you can count by threes and by tens, you can help them eat the correct cookies. Mrs. Monster will eat only the cookies that you put in order counting by threes. Mr. Monster will eat only the cookies that you put in order counting by tens. Mr. and Mrs. Cookie Monster will not bite, but they do get very angry and turn all red if you give them the wrong cookie. Check your work by using the answer key:

ANSWER KEY:	3	6	9	12	15	18	21	24	27	30	33
	10	20	30	40	50	60	70	80	90	100	





9.

Operations

Topic: Addition and subtraction facts

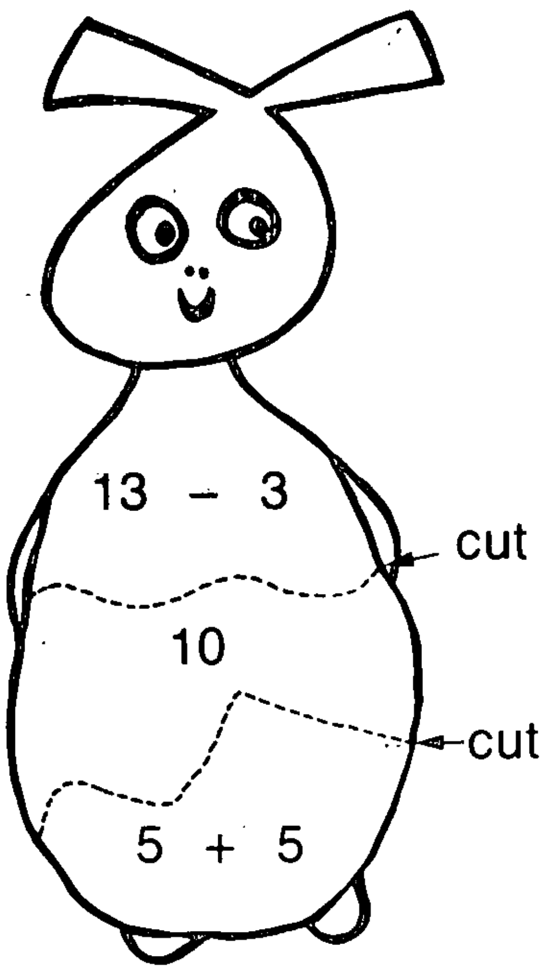
Level: Grades 1–6

Procedure: Print and decorate the rabbit. Laminate. Cut into three parts: subtraction, answer, and addition facts.

For example:

15 − 7	8	1 + 7
12 − 5	7	0 + 7
14 − 5	9	8 + 1
11 − 6	5	2 + 3

Variation: Use for multiplication and division.



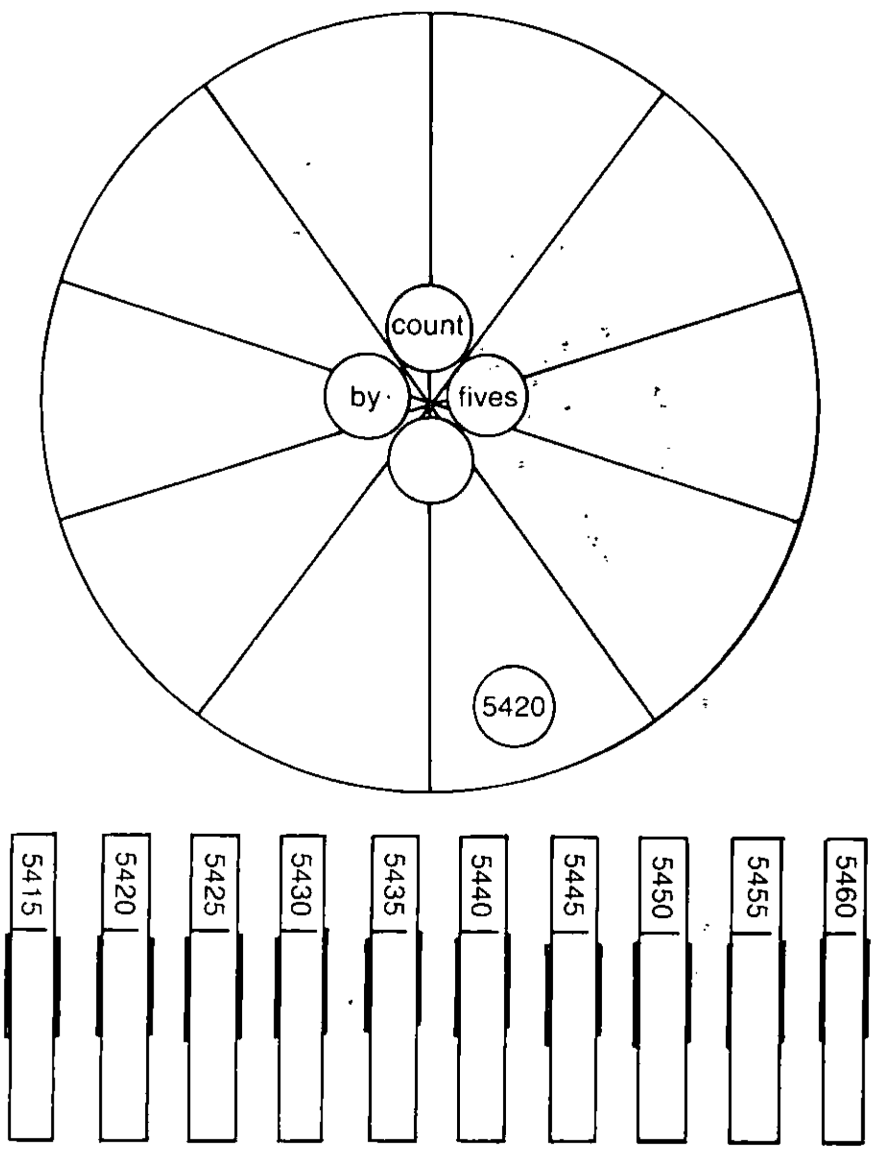
10.

Numeration

Level: Grade 3

Procedure: Counting by fives, pin the clothespins in proper order around the circle.

Variation: Count by ones, tens, twenty-fives, and hundreds.



11.

Place Value

Level: Grade 3

Procedure: Have students place the digits in the appropriate spaces: tens, thousands, ones, and hundreds.

Place Value	t	th	o	h
8 0 0 0	☐	☐	☐	☐
4 6	☐	☐	☐	☐
7 0 2	☐	☐	☐	☐
4	☐	☐	☐	☐
5 1 5 3	☐	☐	☐	☐
3 9	☐	☐	☐	☐
8 6 6	☐	☐	☐	☐
2	☐	☐	☐	☐
9 8 5 4	☐	☐	☐	☐
9 1	☐	☐	☐	☐
3 0 0 6	☐	☐	☐	☐

12.

Expanded Notation

Topic: Place value

Level: Grade 3

Procedure: Punch holes at both sides of the chart where indicated, and tie wool threads at one end. Have students match the other side by pulling the thread.

Variation: Use higher or lower numbers for any grade level.

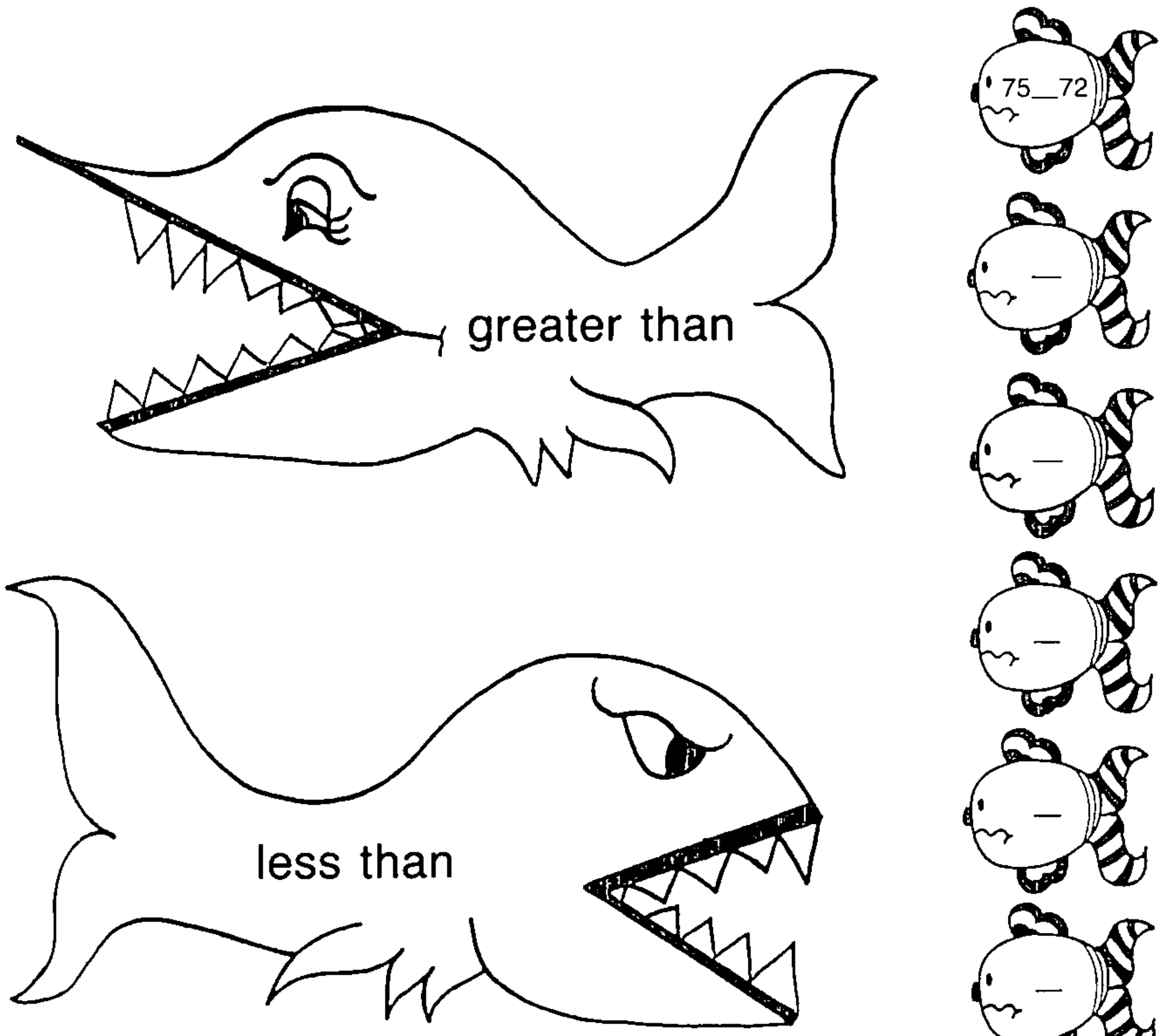
Expanded Numbers

○	9 + 600 + 5000 + 20	7029	○
○	100 + 2000 + 1 + 10	4626	○
○	600 + 6 + 4000 + 20	9770	○
○	9 + 0 + 7000 + 20	1222	○
○	500 + 9 + 6000 + 90	2111	○
○	0 + 9000 + 700 + 70	8132	○
○	20 + 1000 + 2 + 200	5629	○
○	9 + 2000	6599	○
○	30 + 100 + 2 + 8000	3746	○
○	6 + 3000 + 700 + 40	2009	○

13.

Feeding Mr. and Mrs. Jaws

Greater Than and Less Than
(50–100)



14.

Popsicle Match

Topic: Counting

Level: Grades K–1

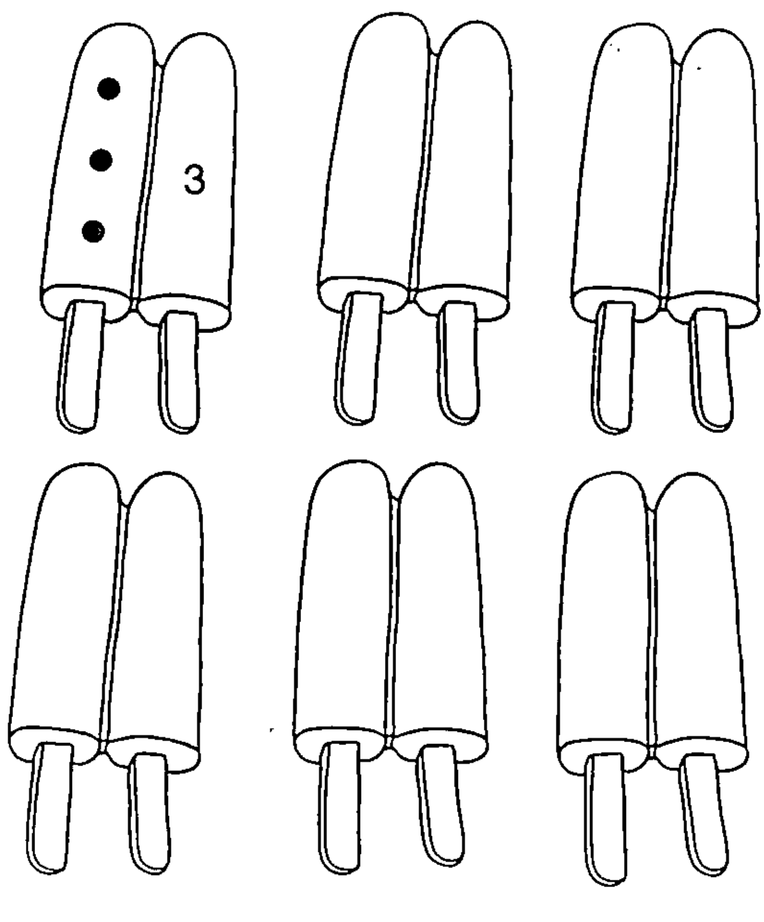
Number of Players: 1 or more

Materials: Popsicle patterns, scissors, envelope for storage

Procedure:

1. Cut out popsicles. Cut the twin sticks into 2 parts.
2. Draw a set on 1 popsicle stick, and write a numeral on the other.
3. Have the students match the set on 1 stick with the numeral on the other stick.

Students may work alone or with a partner.



15.

Bean Drop

Topic: Addition, subtraction, or multiplication

Level: Grades 1–3

Number of Players: 2

Materials: 2 or 3 beans, playing board

Procedure: One student drops the beans on the playing board and adds the numbers on which they fall. The partner checks the sum and takes a turn.

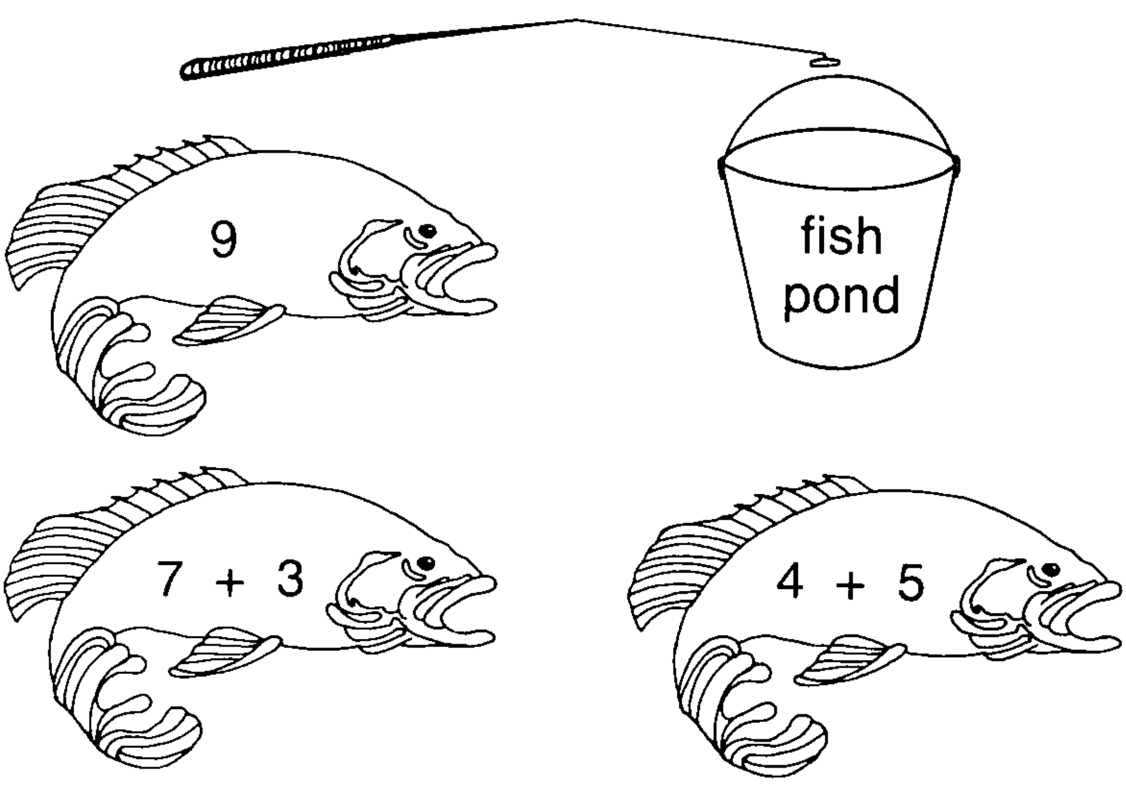
Variations: The same board could be used for subtraction or multiplication.

3	6	5
2	4	9
7	1	8

16.

Gone Fishing

Level:	Grades K–3
Time:	10 minutes
Number of Players:	1 or 2
Objective:	To see who can catch the most fish and set them with the correct answer in a given time.
Materials:	24 fish (more or less), small magnet tied with a string to a stick, 4-litre ice cream pail
Procedure:	Make fish various colors, except for 3 that should be a different color from the others. Laminate. Using a wipe-off felt marker, put answers on the 3 different fish. Put number facts that equal the answers on the remaining fish and attach a paper clip to each one. Place these fish into the pond.
Variations:	Can be adapted for addition, subtraction, multiplication, or division.



17.

101 and Out

Topic: Counting and place value

Number of Players: 2 or more

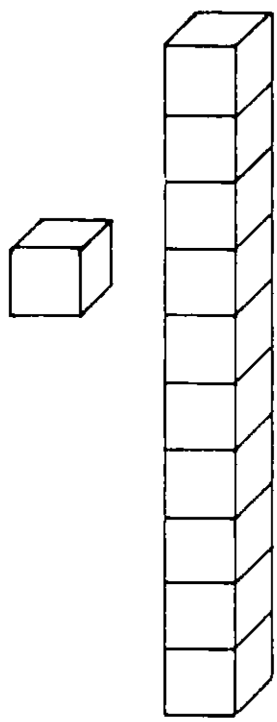
Materials: Number Blox or any other base 10 model, 1 6-sided die, paper, pencil

Procedure: The game begins when 1 player rolls the die and then takes that number of Number Blox, representing either a ones or a tens digit. For example, if a 5 is rolled, the player takes either 5 ones or 5 tens Blox. Players write down their own numbers rolled and keep a cumulative total.

Play proceeds with each player's rolling the die and taking Blox from the central pile. On each turn, a player may decide to take ones or tens Blox, but must take all of a kind in 1 turn.

A player who goes over 100 before taking 10 turns is automatically out. Play continues until every player has had exactly 10 turns.

The player with 100 points, or with the closest number of points under 100, is the winner.



18.

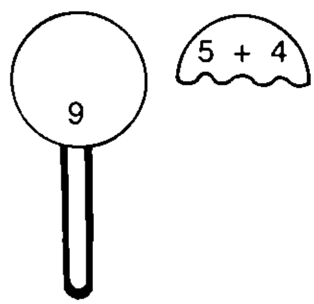
Candy Apples

Strand: Operations

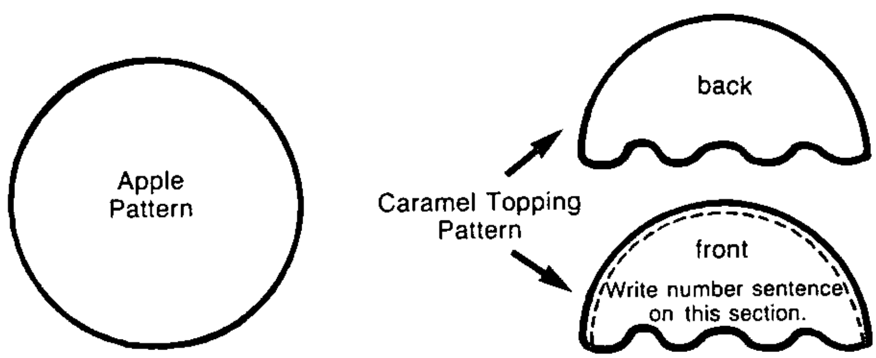
Level: Division I

Objective: To reinforce basic addition and subtraction facts.

Materials: "Apples" on popsicle sticks, with one sum written on the face of each apple; "caramel toppings," with a number sentence written on the face of each.



Procedure:



1. Cut out red, green, or orange circles. Write answers on the bottom half of these circles. Laminate. Staple each circle to the top of a popsicle stick.
2. Cut out the topping pieces, and glue them together along the dotted line shown. Write a number sentence on the front of each topping. Laminate.
3. Using an Exacto knife, slit the laminating film along the bottom edge of the toppings so that the toppings will slide over the apples.

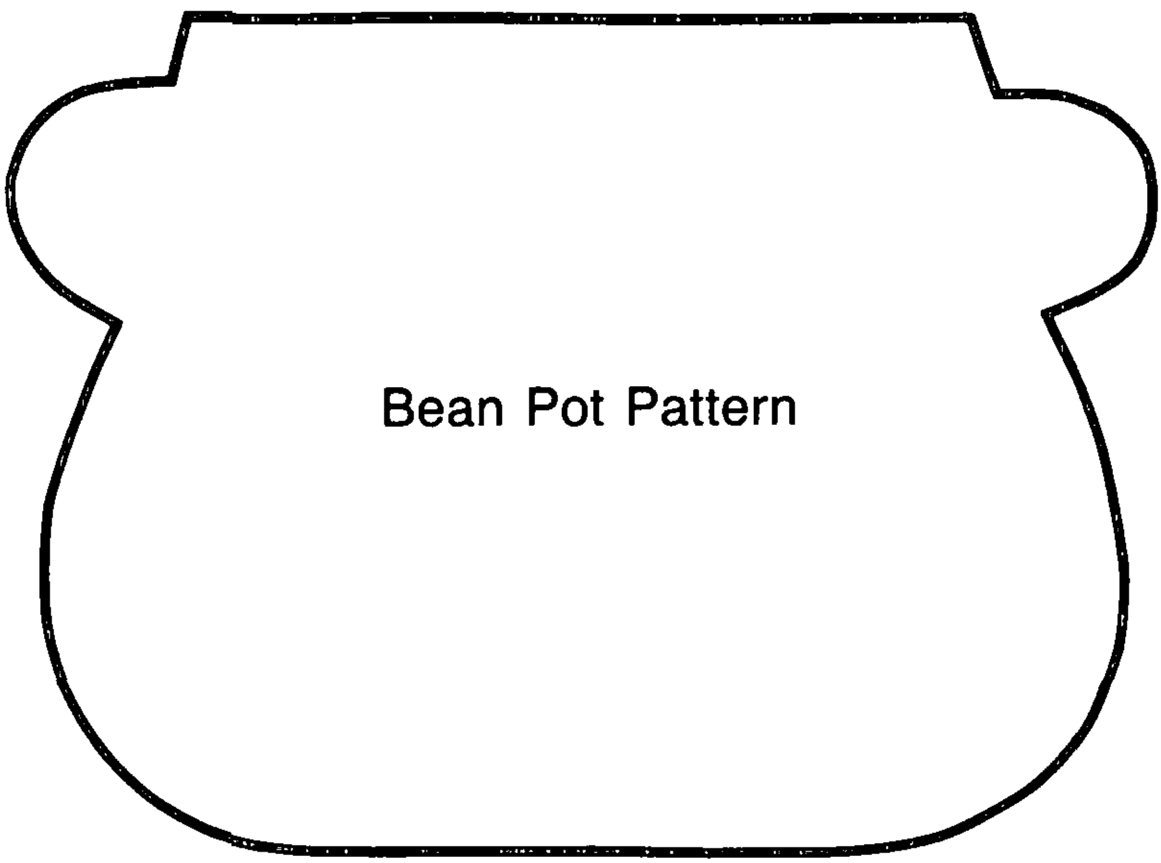
The caramel toppings are slipped over the correct sum or minuend. This activity can be used with a circus, nutrition, Halloween, or fall theme.

Variation: The operations can be changed to meet the needs of the students.

19.

Bean Pots

Topic:	Number concepts
Strand:	Counting, learning invariance of numbers, visual memory
Level:	ECS–Year 1
Time:	10 minutes
Type of Activity:	Individual or small group (3)
Materials:	Bean pots numbered 0–10, kidney beans
Procedure:	Have students place the appropriate number of beans on each pot and discuss results with the teacher.



20.

Mittens in the Basket

Level: Division I

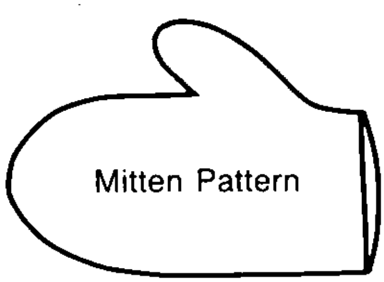
Materials: Several colorful "mittens," "baskets" mounted in a file folder

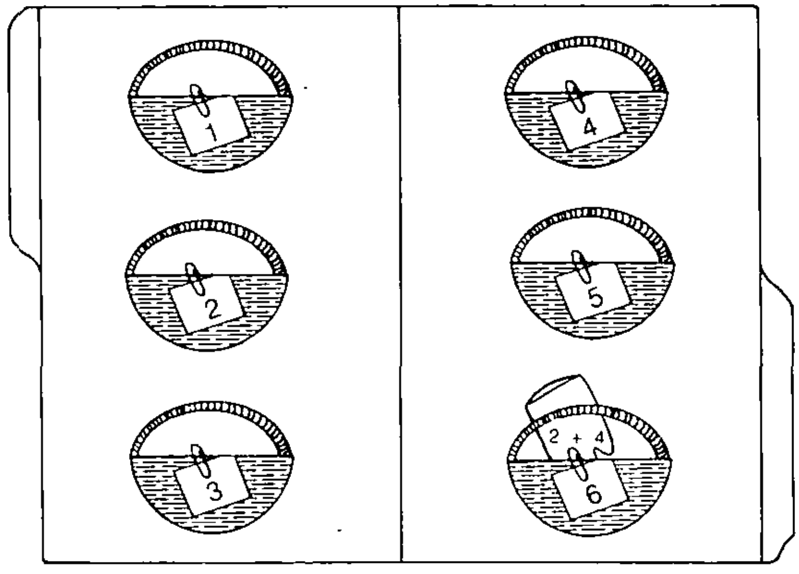
Procedure: Cut out several mittens and write number sentences on them. Laminate. Use library card pockets to make baskets, or design your own. Glue the baskets into a file folder. Use a paper clip to attach the required answer to each basket, so that answers may be changed according to the students' needs.

Children use counters, if desired, to solve the math sentences written on each mitten. Then they place the mitten in the basket numbered with the corresponding answer.

Once all the mittens have been put into their baskets, it's easy to check students' work, or include an answer card to make it self-correcting. The number sentences could also be copied into a math "tub" notebook at this time.

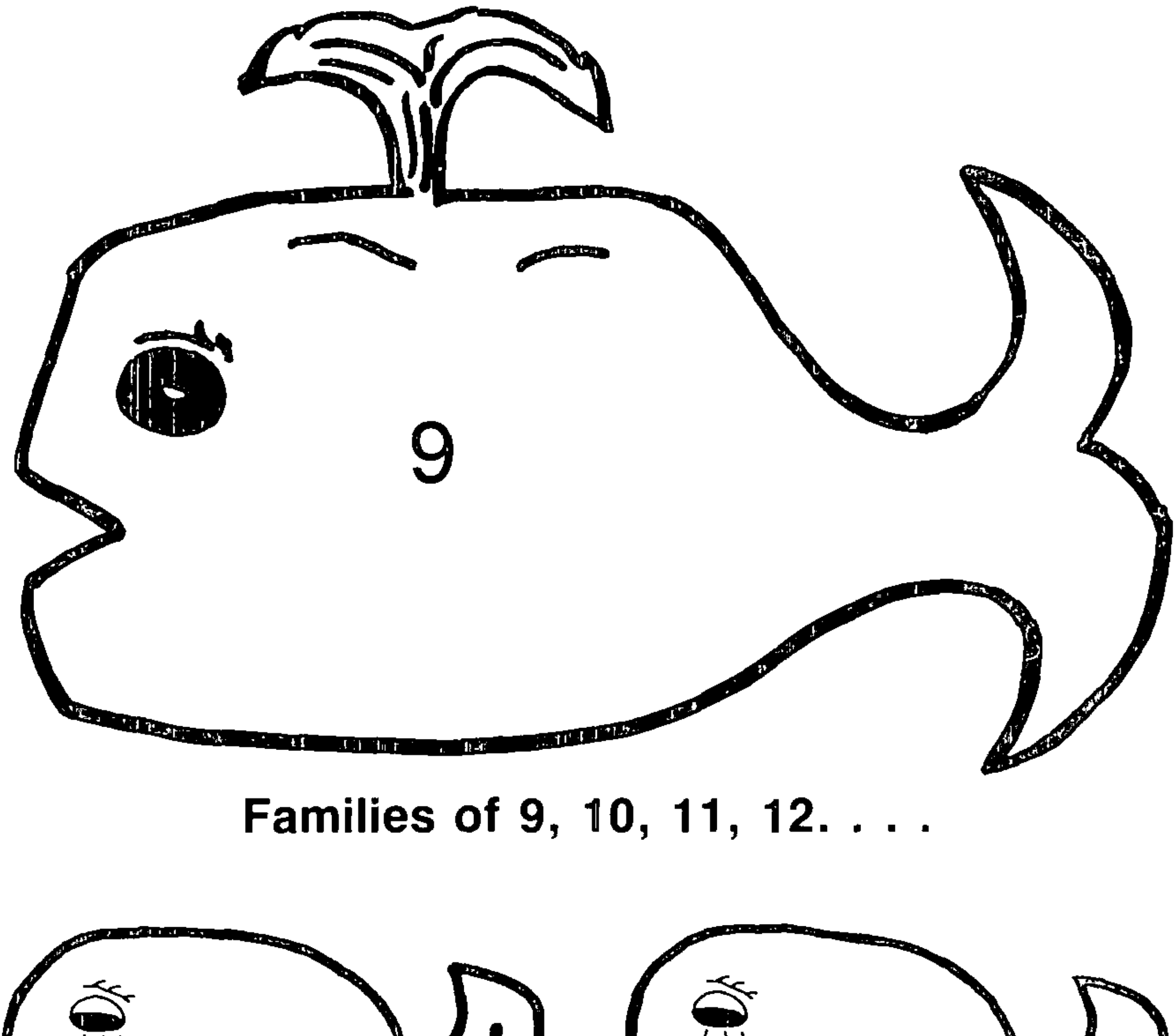
This activity was used during a nursery rhyme and winter theme.





21.

Family Whales



Families of 9, 10, 11, 12. . . .

22.

Mini-Paper Chains

Topic: Numeration

Strand: Addition, subtraction, writing numerals

Level: Grades 1–3

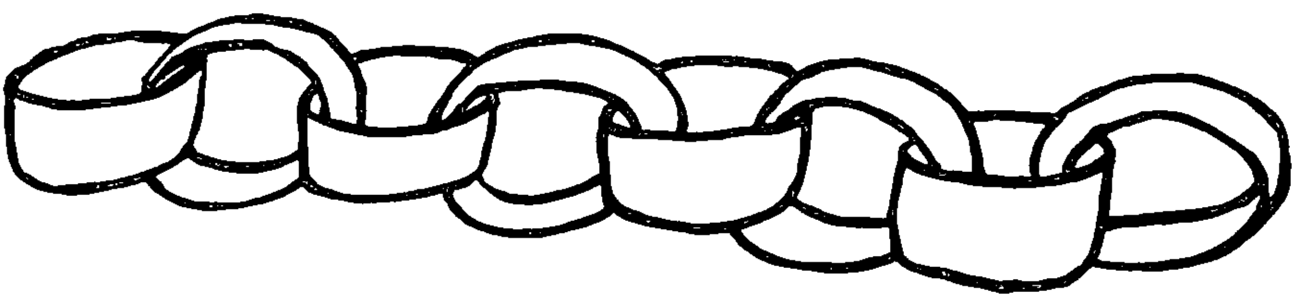
Type of Activity: Individual or small group

Materials: Christmas wrapping paper cut into mini-strips, with math facts on the back; glue; pencils; erasers

Procedure: Have the students—

1. choose 12 strips to make a paper chain.
2. answer the questions on the back of the paper strips and get the teacher to check their answers.
3. glue the paper strips together to make a paper chain.
4. link all the chains to make one big chain to decorate a Christmas tree.

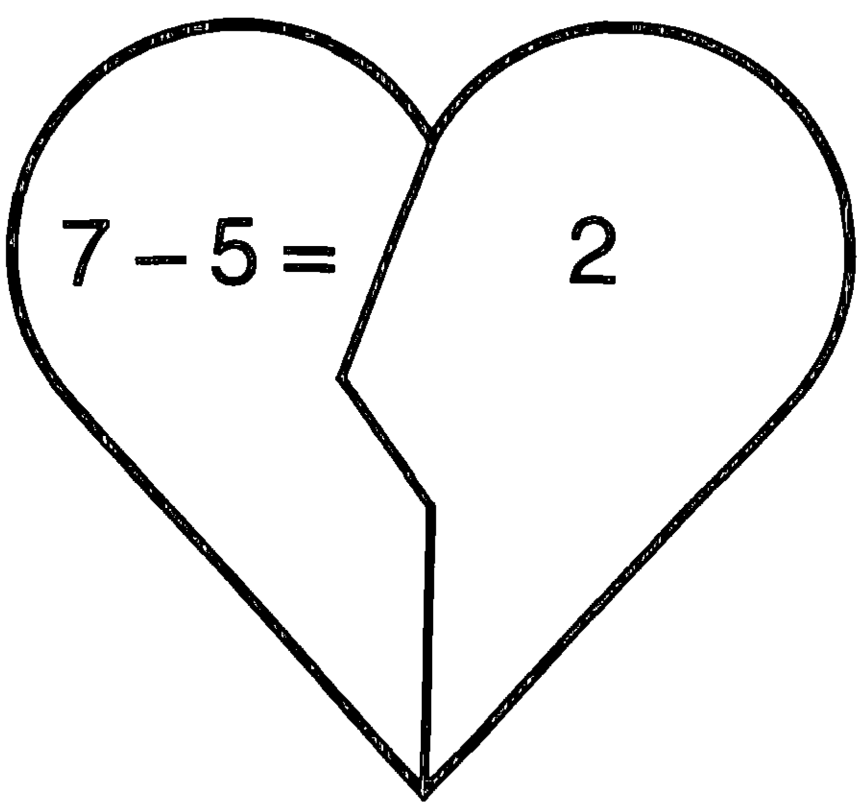
Variation: Have the students choose 7 paper strips and follow procedures 2 and 3 above. Cut out 2 bells (use templates for tracing, if desired). Decorate the bells with paper chains.



23.

Mend the Broken Hearts

Topic:	Addition and subtraction or multiplication and division
Level:	Grades K–3
Time:	5–10 minutes
Number of Players:	2
Objective:	To see who can mend the most hearts.
Materials and Preparation:	Make 20 hearts (more or less), 14 cm x 16 cm, from red construction paper. Laminate, then cut each one apart differently. Write a question on one part of each heart; the answer on the other. Use a felt marker with ink that can be easily wiped off, so that questions can be changed frequently.



24.

Making a 5-Pointed Star

The regular 5-pointed star has been a popular ornamental figure for centuries. It was a symbol of the ancients. Old drawings show that the star was a favorite of magicians and astrologers. In more recent times, it served as a symbol for the Christians. For some peoples, it has played an important part as a talisman against different forms of misfortune; some have put it on baby cribs to fend off evil; others have painted it on farm buildings to protect the animals. Today this figure is seen in our military rating marks, on police officers' badges, and in decorative designs.

There is no direct relationship between the shape of the stars that have been described and the stars in the sky, except that the heavenly stars seem to be surrounded by rays of light that suggest the 5-pointed geometrical star.

The diagrams and directions below illustrate how to make a star by folding and cutting a sheet of paper.

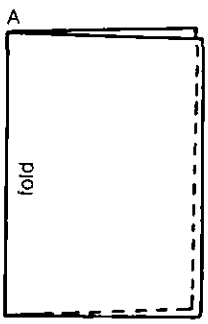


Figure 1.

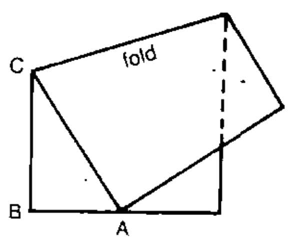


Figure 2.

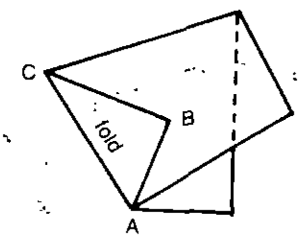


Figure 3.

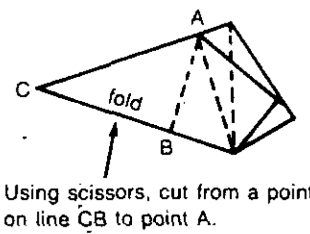


Figure 4.

1. Fold, from left to right, the top of a sheet of regular typing paper. Crease the fold, as in Figure 1.
2. Place the corner marked A on the centre of the opposite edge, and fold flat, as in Figure 2.
3. Fold flap over from point B and crease, as in Figure 3.
4. Now, fold A over to the top edge and crease, as in Figure 4.
5. Using scissors, cut from a point on line CB through point A. Open up the cut-off part, and you will have a finished star.

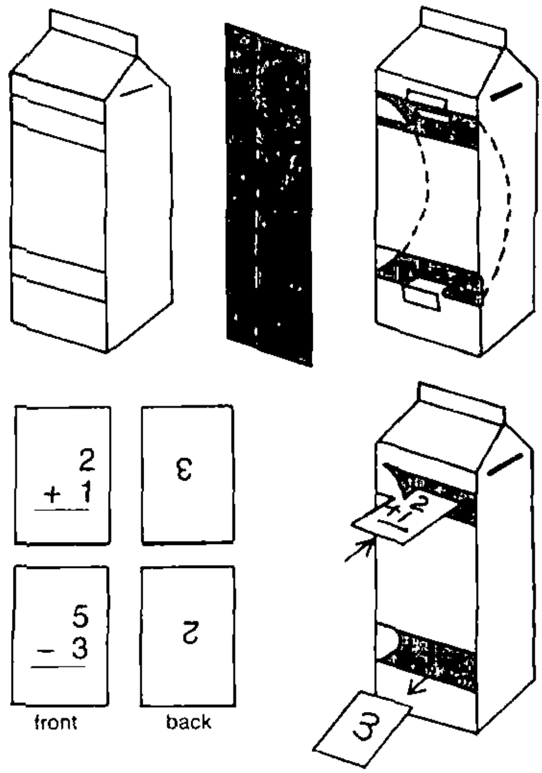
Experiment with the stars made in the above manner by starting your cuts at different points on line CB and by cutting to various points along line CA. Can you explain how the different cuts change the size and shape of the resulting stars?

25.

Math Computers

Materials: 1-litre milk carton; strip of tag, 7 cm x 25 cm; tape; computer cards, 6 cm x 10 cm; flash cards +, −, ×, ÷

Construction:



Procedure: Children will love practising math problems with their own mini-math computer. Decorate a milk carton or a detergent box like a computer. On one side of the carton, cut 2 slots: one at the top and one at the bottom. Next, cut out a strip of oaktag to make a slide, as shown. (Push the strip in the top slot, down the container, and out the bottom slot.) Tape the top of the strip to the top slot. Tape the bottom of the strip to the bottom slot.

Cut several "computer cards" out of oaktag. Write math problems on one side of the cards and the answers upside down on the reverse side.

To play, each child chooses a problem, drops it into the top slot of the computer, and tries to say the answer before the card slides out the bottom. (The card will flip over inside the box to reveal the correct answer as it comes out the bottom.)

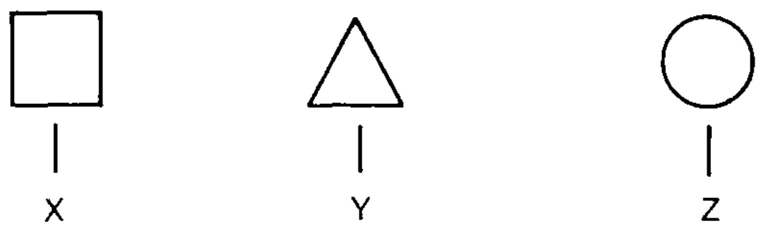
Variation: You can also use the computer to practise vocabulary words by taping a picture to one side of the card and writing the corresponding word on the flip side.

26.

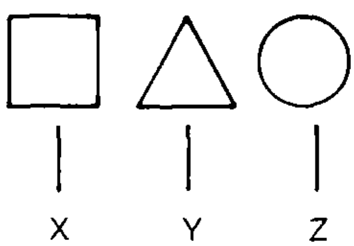
Snap Number System

How would you like to develop your own number system?

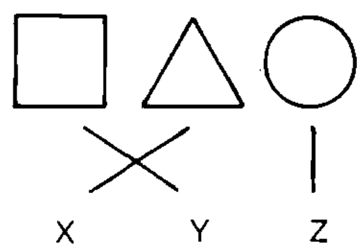
Begin by selecting 2 arbitrary sets of symbols, and set up all the possible one-to-one correspondences between them.



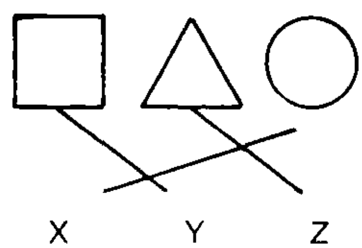
How many combinations are there?
Did you get the combinations shown below?
Students like to make up names for each symbol, for example, Egor.



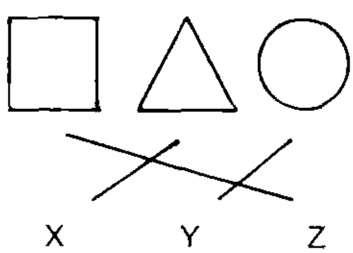
Egor



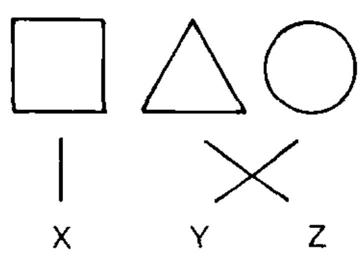
Zwak



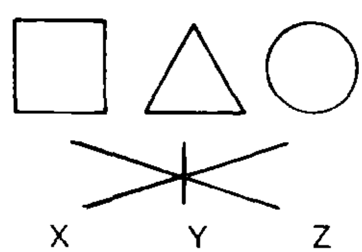
Bogo



Moto



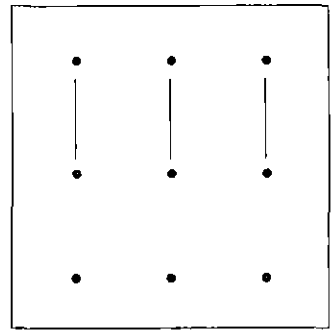
Flipo



Tik

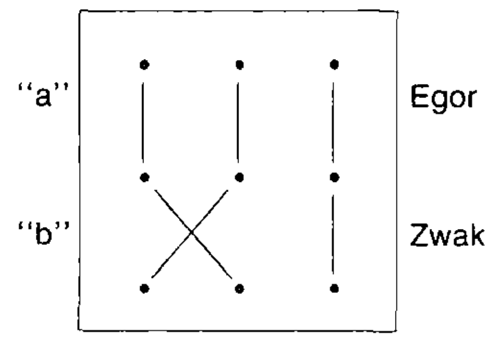
The order of the patterns and the labels (or symbols) of the elements may be chosen arbitrarily.

The patterns of the above elements can be represented on a square "snapboard" consisting of 9 nails in a pattern as shown on the diagram to the right. Rubber bands can be used to represent the patterns as shown on the diagram.



"a"—Egor

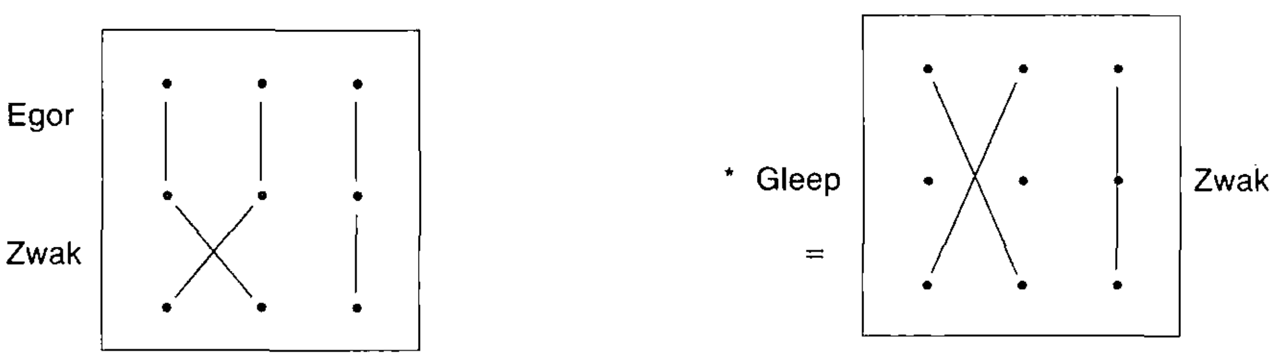
After the elements have been defined, an operation must be agreed upon. The second element may be placed on the snapboard by simply pulling the rubber bands over the third set of nails. Therefore, the position or relationship of the 2 elements "a" and "b" will be shown as on the diagram to the right. Students also like to make another name for "equals," for example, Gleep.



This emphasizes that our system has resulted from people's agreeing upon terminology.

Our operation between the 2 elements merely consists of "snapping" the rubber bands off the middle set of nails. The rubber bands will form a new pattern, which must be one of the 6 defined elements.

Can you think of a symbol and a name for the new operation? Let's arbitrarily call it "snap" and symbolize it as *.



Egor * Zwak Gleep Zwak

In like manner, the operation of all the elements can be calculated.

What would be the pattern of elements on the following matrix as a result of the operation snap?

*	E	Z	B	M	F	T
E						
Z						
B						
M						
F						
T						

Are there any similarities between this pattern and those of basic operations on whole numbers such as the identity and commutative or associative properties?

Questions for Further Study

1. How would the matrix be affected if we begin with 2 sets, each containing 4 or more elements?
2. What is the relationship between the number of elements and the number of one-to-one correspondences?
3. Is it necessary for each set to have the same number of elements?
4. Could the operation snap be used on 3 or more elements at the same time?
5. Can you formulate any properties of the operation snap on the given elements?
6. Why should "snap" be any different from any of the basic operations?
7. Does "snap" help us to understand the properties of whole numbers? Explain.
8. Could the game "snap" have any practical applications?
9. Why has man agreed upon only the 4 basic applications?
10. What is an axiom? Cite a case in which mathematicians have used axioms. Why are axioms useful?
11. Can you create a similar game with different symbols and a different operation?
12. Could people have developed other number systems and algorithms? Explain.

27.

"The Remainder" Game

Topic: Division of whole numbers

Number of Players: 2–4 players on each board

Materials: Game board (1 of the 2 included); package of cards, 3 each of cards numbered 2–9; paper; playing markers; pencil

Procedure:

1. Shuffle the deck of cards.
2. Each student draws a card. The student with the highest number goes first.
3. The cards are returned to the deck and the deck is reshuffled.
4. The first student draws a card and divides the number on the card into the number on the board that his or her playing piece occupies.
5. The remainder is the number of squares the player moves.
6. Other players check the drawing player's result. If the drawing player is wrong, that player must return to start, but if the challenger is wrong, then the challenger must return to start.
7. Play continues until one player reaches home. This player is the winner.

2	3
4	5
6	7
8	9

34	77	64	50	39	13
85	98	16	5	9	93
55	84	HOME		32	86
14	130			8	74
37	115	70	59	38	18
82	64	35	17	20	START 19

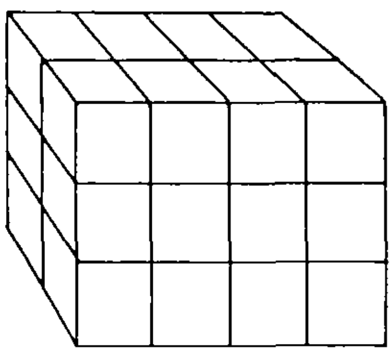
100	538	192	6783	500	325
4760	784	8320	536	107	7314
7413	460	HOME		824	122
966	948			369	576
249	6322	362	2391	643	5729
3116	8021	444	527	624	START 135

28.

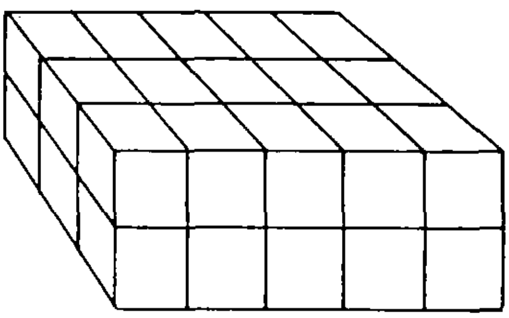
Volume

Make up 3 rectangular solids using cubic centimetre multi-link blocks.

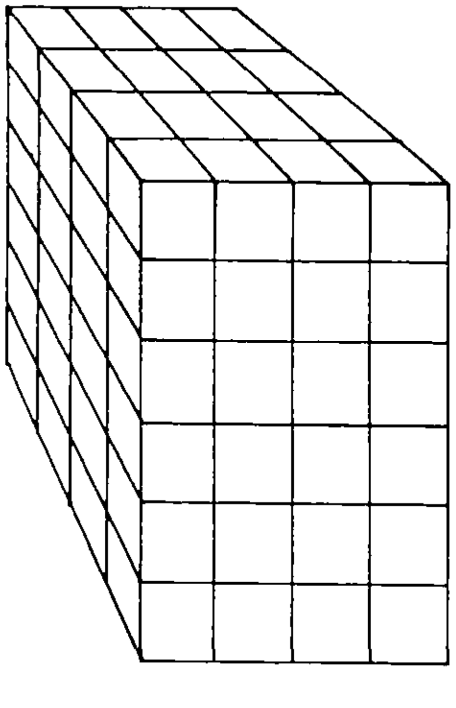
(a)



(b)

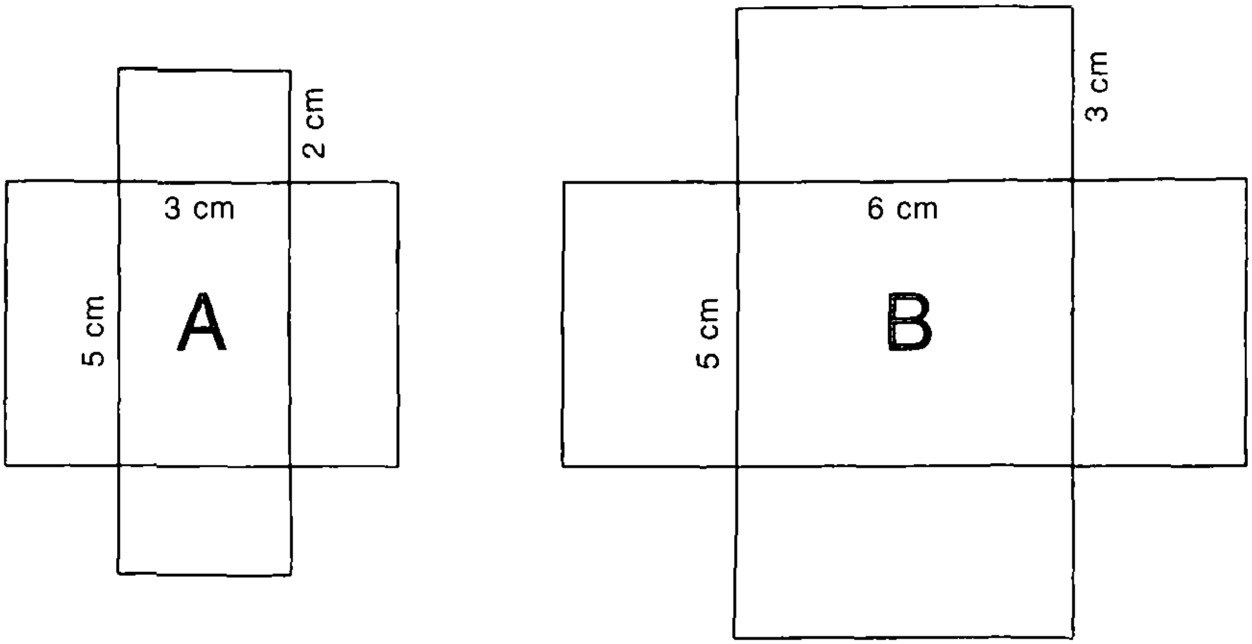


(c)



(d) Have students develop blocks of specific dimensions. For example: 4 in a row, 3 rows, 5 high.

Make 2 boxes. Let students fill the boxes with cubes.



Make boxes with the dimensions given or decided upon by the class. Suggested dimensions:

Box C	12 cm x 8 cm x 5 cm
Box D	6 cm x 6 cm x 10 cm
Box E	6 cm x 9 cm x 14 cm

Ask the following questions: How many cubes in a row? How many rows? How many layers? How many cubes in total?

Questions

1. Count the number of cubes in:
   (a) __________ ?
   (b) __________ ?
   (c) __________ ?

2. Fill boxes A and B with cubes. How many cubes do you need for:
   Box A __________ ?
   Box B __________ ?
   What is the volume of Box A? __________
   What is the volume of Box B? __________

3. What is the volume of Box C? __________
   What is the volume of Box D? __________
   What is the volume of Box E? __________

4. What would be the volume of the box that has:
   (a) 5 cubes in one row
   5 rows
   5 layers ______
   (b) 8 cubes in one row
   7 rows
   4 layers ______
   (c) 11 cubes in one row
   8 rows
   10 layers ______

29.

Product Squares

Topic: Multiplication and division using a calculator and estimation

Level: Grades 4–5

Number of Players: 2

Materials: 2 calculators, markers of 2 different colors, game board

Procedure: Cut the game board from the 2 number strips. Separate the 2 number strips, and give 1 to each player. The students take turns choosing 2 numbers from the number strips (numbers in circles) whose product is a number on the game board. Students must estimate aloud the product. They must then multiply using the calculator, and place a marker over the correct product on the game board. The first player to get 4 markers in a row is the winner.

Players are encouraged to correctly estimate products so that they will be the first to cover a row of numbers or to use a correct answer to block an opponent from winning.

66	336	84	42
777	98	407	128
56	592	88	518
77	112	126	48

7	11	6	37	21	16	14	8
7	11	6	37	21	16	14	8

30.

Divide It, Match It

Topic: Division facts

Level: Grade 4

Number of Players: 2–4

Materials and Preparation: Prepare 52 7 1/2 cm x 12 1/2 cm cards so that 9 of them display a division fact whose quotient is 5, 9 of them display a division fact whose quotient is 6, and so on for quotients 7, 8, and 9. Seven cards have no division facts, but display the words "Math Wizard."

Procedure: Each player is dealt 5 cards; the remaining cards are placed face down on the centre of the table. The first player puts 1 division fact card face up on the table and gives the answer to the problem shown. The next player must place a card with the same quotient on the first card. If unable to match the quotient, the player places a Math Wizard card on top, and then a card with a different quotient for the next player to match. If unable to make either move, the player draws from the deck until able to play.

The first player with no cards wins the game.

Sample Facts for Card Development

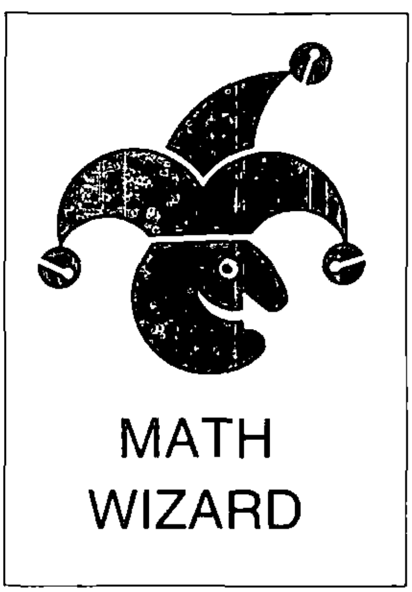
5 as quotient: $5 \div 1$, $10 \div 2$, $15 \div 3$, $20 \div 4$, $25 \div 5$, $30 \div 6$, $35 \div 7$, $40 \div 8$, $45 \div 9$

6 as quotient: $6 \div 1$, $12 \div 2$ . . . $54 \div 9$

7 as quotient: $7 \div 1$, $14 \div 2$ . . . $63 \div 9$

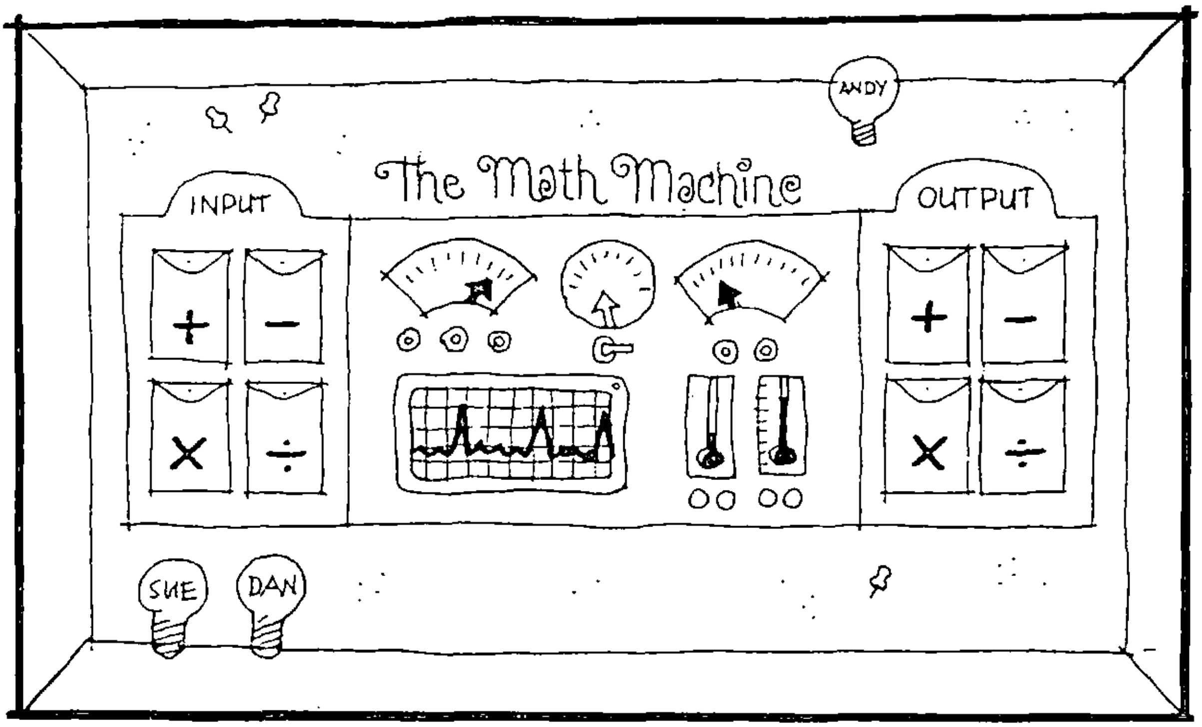
8 as quotient: $8 \div 1$, $16 \div 2$ . . . $72 \div 9$

9 as quotient: $9 \div 1$, $18 \div 2$ . . . $81 \div 9$

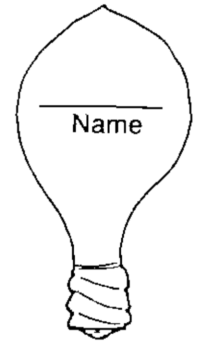


31.

Bulletin Board Idea



1. Cut 2 small rectangles and 1 long one from construction paper. Label the 2 small ones *input* and *output*, and place 1 at each end of the long rectangle. Staple, tack, or tape envelopes large enough to hold standard-sized worksheets to the input and output rectangles. Decorate the centre rectangle with various paper dials, knobs, lights, screens, and switches to make it truly a "math machine." You may wish to include an answer key envelope near the output rectangle.
2. On a regular basis, place in the envelopes new math worksheets providing review and reinforcement, new skill preview, extra-credit and spare-time activities, or a combination of these.
3. Students should individually select worksheets from the input envelopes, complete the problems or activities, and, if answer keys are provided, check their work before placing the finished sheets in the proper output envelope.
4. Encourage students to create a border by placing personalized light bulbs on the board for each perfect paper.



Light Bulb Pattern

Variation

Instead of input worksheets emphasizing 4 different mathematical operations, use input material dealing with a single skill or operation at varying levels of difficulty.

32.

Expanded Notation Concentration

Level: Grades 3–4

Time: 10–15 minutes

Number of Players: 2 or more

Materials and Preparation: Mark 20 (or more) cards 11 cm × 4 cm (must be an even number) in pairs using whatever place value is being studied. For example:

321	8104	64
(3 × 100) + (2 × 10) + (1 × 1)	(8 × 1000) + (1 × 100) + (0 × 10) + (4 × 1)	(6 × 10) + (4 × 1)

Laminate cards.

Procedure:

1. Cards are spread out, face down, either in a random or an ordered fashion.
2. Each player chooses a card. The player who chooses the one with the highest numerical value begins the game.
3. A player turns over 2 cards. If there is a match, the pair is kept by the player, who may choose again. If there is no match, the play moves on to the next player.
4. Play continues until all cards have been matched.
5. Players count the number of matched pairs that they have. The person who has the most pairs is the winner.

Variations: This "concentration" format can be extended to many other concepts, such as fractions, addition, subtraction, and symmetry.

33.

Math Game Boards

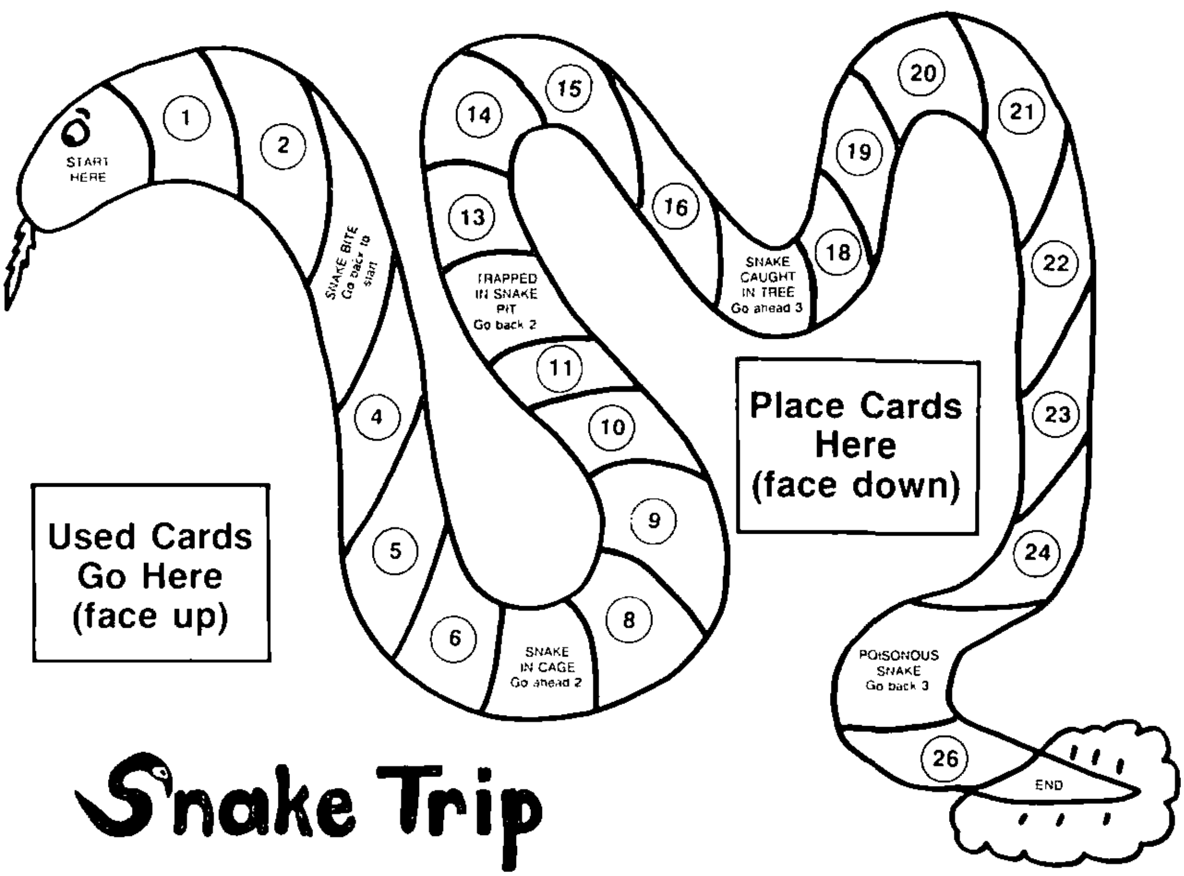
Objective: To make a game board that students will use to reinforce basic addition, subtraction, multiplication, or division skills.

Materials: 1 piece manila tag, felts or pencil crayons, small playing cards

Procedure:

1. Have students design their game board and decorate it using such items as pencil crayons, felts, and stickers. (A sample game board is shown below.)
2. Have students write, on small playing cards, basic facts involving either addition, subtraction, multiplication, or division. For example: $2 \times 6 =$ ______.
3. Have students design the rules of the game and write them on the back of the game board.
4. If you wish, laminate the game board and cards so that they will last longer.

Variations: Vary the procedures according to the needs of individual students.



34.

Equivalent Fractions

Level: Introductory Grade 7

Time: 1 class period (40–50 minutes)

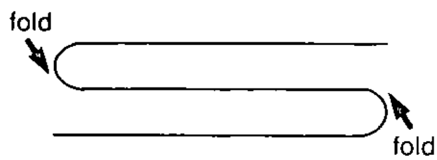
Objective: To write sets of equivalent fractions.

Prerequisite Skills: Representation of a fraction by illustration; identification of the fraction form of a number

Materials: Precut lengths (1 metre) of adding machine tape

Procedure:

1. Mark the zero (0) point at the left end of the tape and the one (1) at the right end of the tape.
2. Fold tape in half lengthwise.
3. Mark crease with 1/2. Mark ends as 0/2 and 2/2.
4. Fold into thirds (use "S" shape shown).
5. Mark 0/3, 1/3, 2/3, and 3/3 on new folds.
6. Fold into quarters.
7. Mark 0/4, 1/4, 2/4, 3/4, and 4/4.



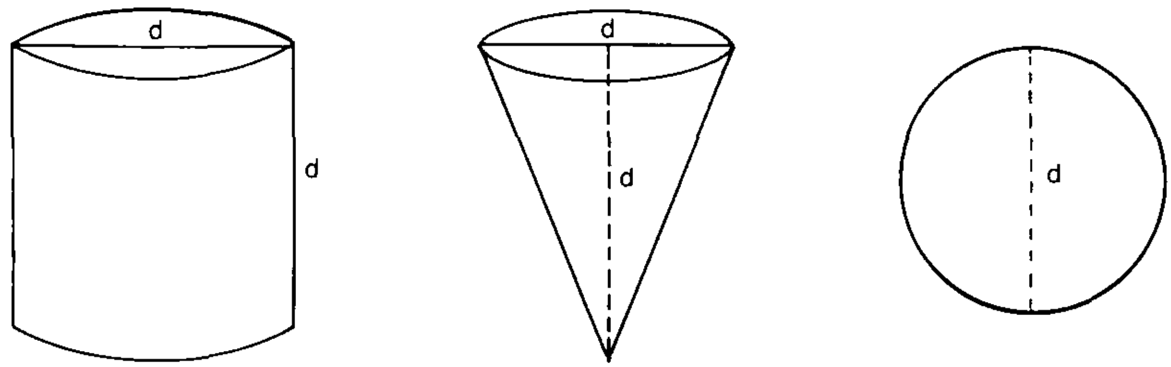
8. Have students write, on a separate paper, fractions that name the same point (for example, 1/2 = 2/4, 0/2 = 0/3 = 0/4, and so on).
9. Continue folding in this pattern to get sixths, eighths, and twelfths.
10. Have students mark the fractions on the tape after *each* fold, and list on paper all true statements. For example, 1/2 = 2/4 = 3/6 = 4/8, 1/4 = 2/8 = 3/12, 1/3 = 2/6 = 4/12.
11. Write all fractions that appear on the 1/4 fold.
12. Have students guess what other fractions would appear on this fold and state why they would appear at this point.
13. Establish a general rule for writing equivalent fractions from the basic fraction.
14. Test the rule by examining other creases.
15. Ask students to imagine that they are folding paper into sixteenths. Determine which sixteenths would appear at the 1/4 mark, 1/2 mark, and 1/3 mark.

Suggestions:

1. Refolding to identify points may be necessary.
2. Strips should be saved for subsequent activities (ordering fractions, decimals, and percents).
3. Less able students and/or classes may wish to use only 1/2, 1/4, and 1/8.

35.

Volume of a Sphere



1. Place the sphere into the cylinder.
2. Use the cone to pour water to fill the cylinder.
3. The experiment shows that:

Volume of cylinder $= \pi r^2 h$

$= \pi r^2 \times 2r$

$= 2 \pi r^3$

Volume of cone $= \frac{1}{3} \pi r^2 h$

$= \frac{1}{3} \pi r^2 \times 2r$

$= \frac{2}{3} \pi r^3$

Therefore:
Volume of sphere = Volume of cylinder − Volume of cone

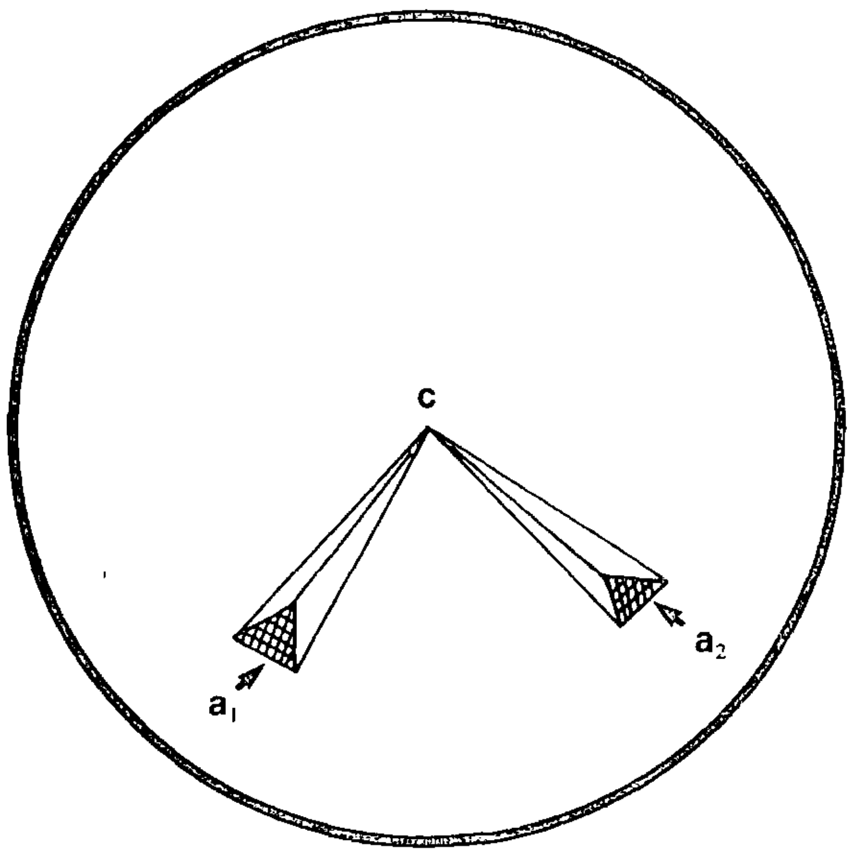
$= 2 \pi r^3 - \frac{2}{3} \pi r^3$

$= (2 - \frac{2}{3}) \pi r^3$

$= \frac{(6-2)}{3} \pi r^3$

$= \frac{4}{3} \pi r^3$

Surface Area of a Sphere



Volume of pyramid = $\frac{1}{3}$ $a_1 \times r$

If the sphere is made up of an infinite number of pyramids, then

$$\frac{1}{3} a_1 r + \frac{1}{3} a_2 r + \ldots = \frac{4}{3} \pi r^3$$

$$\frac{1}{3} r (a_1 + a_2 + a_3 + \ldots) = \frac{4}{3} \pi r^3$$

But $a_1 + a_2 + a_3 + \ldots$ is the surface area of sphere.

Then $\frac{1}{3}$ r (A) = $\frac{4}{3} \pi r^3$

$$A = \frac{4}{3} \pi r^3 + \frac{1}{3} r$$

$$A = \frac{4}{3} \pi r^3 \times \frac{3}{r}$$

$$= 4 \pi r^2$$

36.

Area of a Circle

Objective: To determine the formula for the area of a circle.

Prerequisite Skills: Familiarity with the area of a rectangle; the terms radius, circumference, diameter, semi-circle; the formula $C = \pi d$

Materials: Colored poster board, glue, scissors

Procedure: Give each student a circle, on which the centre should be clearly marked. Have students:

1. Draw a diameter. Mark the 2 radii on the diameter, and mark one-half the circumference ($\frac{c}{2}$) on each semi-circle.
2. Cut along the diameter. Put the two semi-circular regions together so that the edges coincide.
3. Beginning at the centre each time, cut along many radii, coming very close to the edge of the circular region but not cutting the edge. This results in a string of pie-shaped wedges (about 10).
4. Paste 1 string of pie-shaped wedges on a different piece of colored paper to suggest a rectangular region. Note that one-half of the rectangular region is covered with the semi-circular region.
5. Paste the second string of pie-shaped wedges on the first so that the entire rectangular region is covered and the wedges are not overlapping.

Since the circular region has now been adapted to a rectangular region, it is possible to estimate the area of a circle. The area of the circular region is one-half the circumference times the radius.

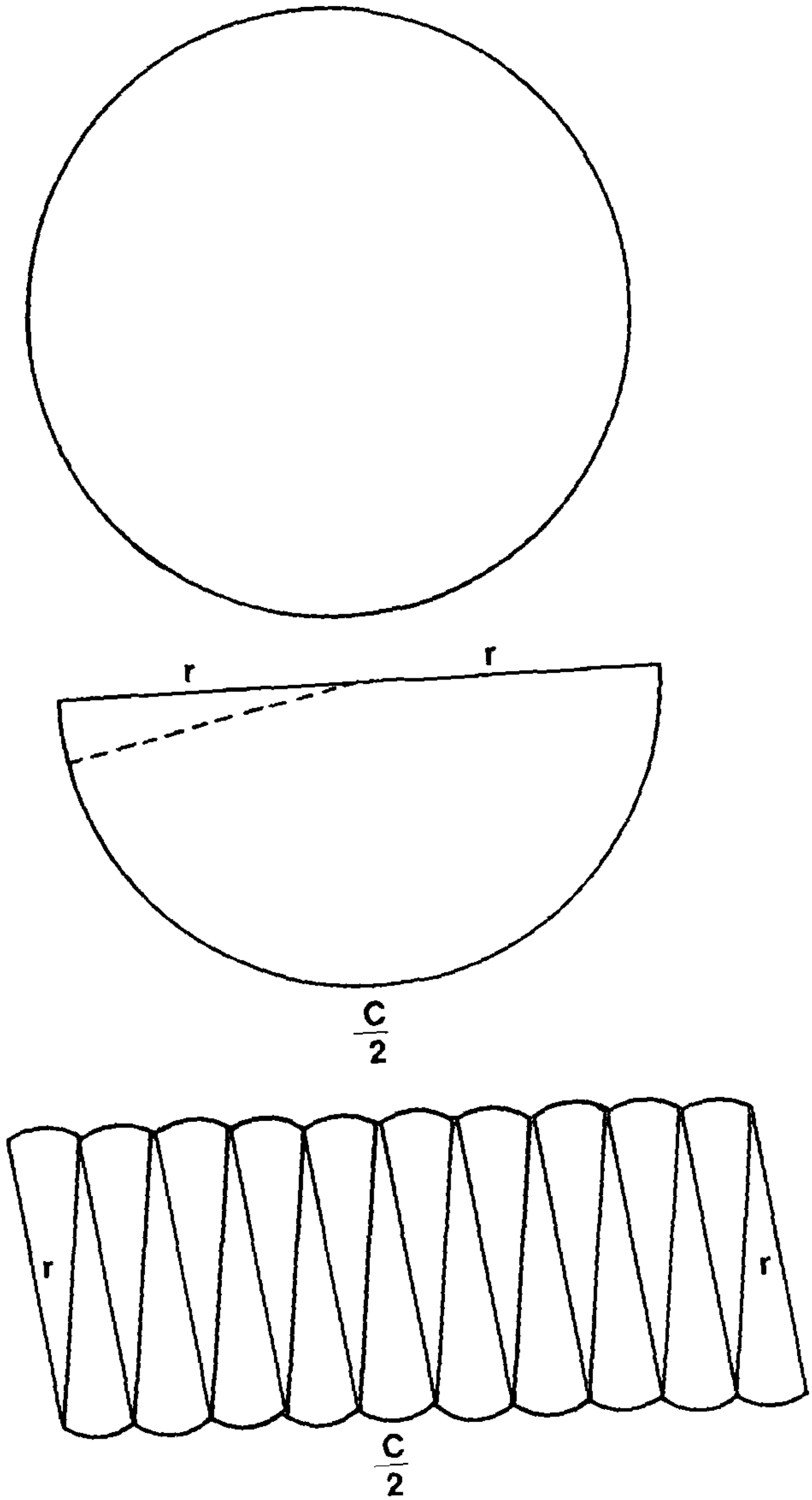
The formula for the area of a circular region is:

$$\text{Area} = \frac{1}{2} \times C \times r$$

$$C = 2\pi r$$

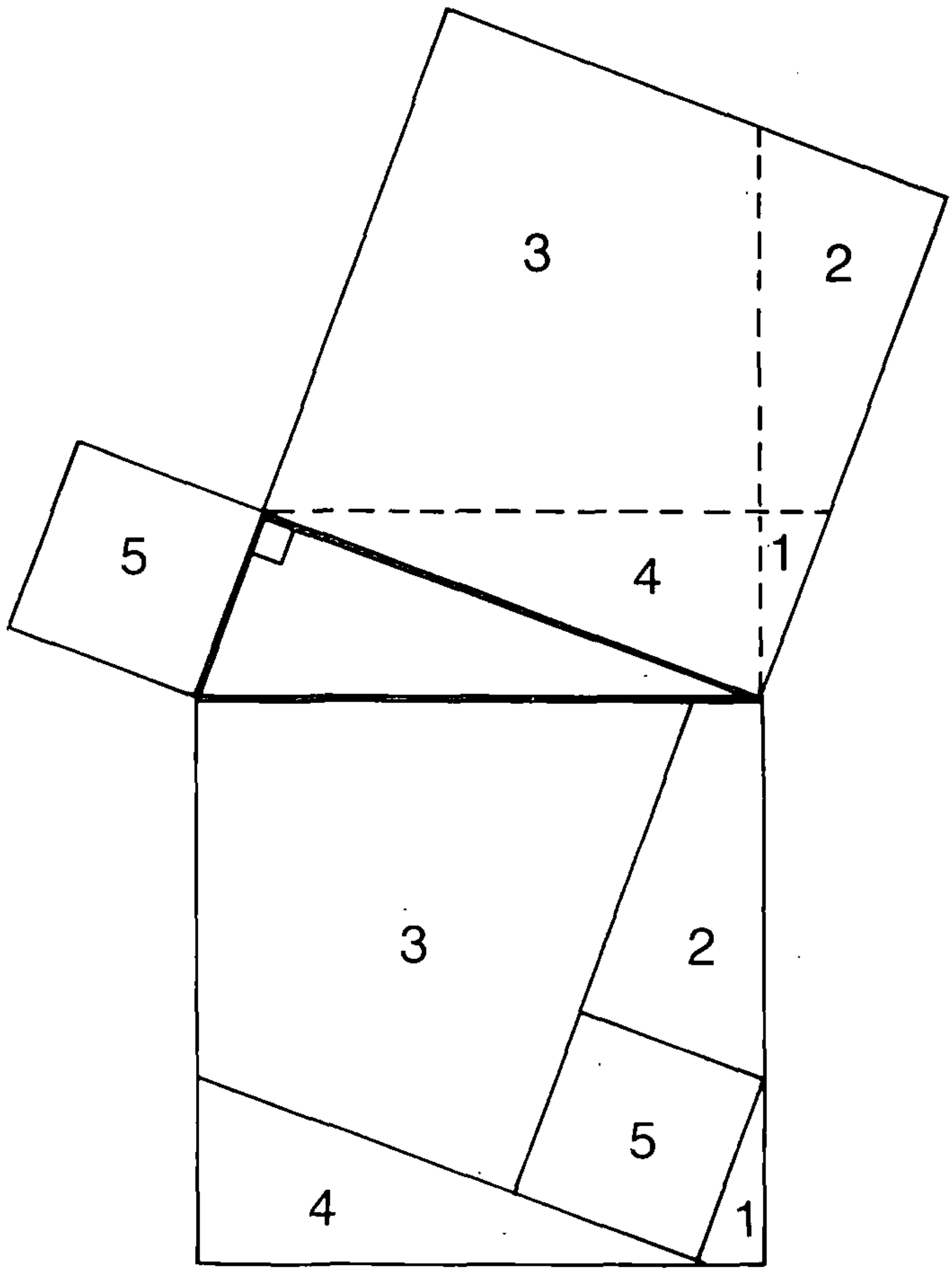
$$A = \frac{1}{2} \times 2\pi r \times r$$

$$A = \pi r^2$$


r
r
C/2
r
r
C/2

37.

Pythagorean Theorem



38.

Addition and Subtraction of Integers

Addition of Integers

Level: Introductory Grade 8

Time: *1 class period (40–50 minutes)*

Objective: To generate an understanding of the addition of integers using number lines (slide rule).

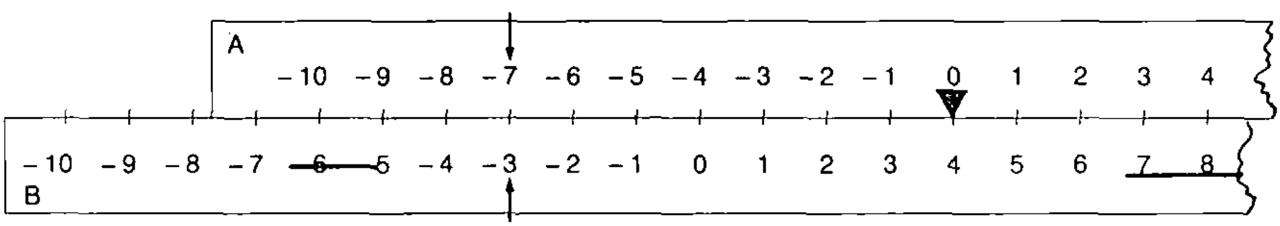
Prerequisite Skills: Placement of integers on number lines; directional meaning of positive and negative signs; identification of addends

Materials: 1 set of numbered scales for each pair of students, scissors, 1 set of numbered scales on transparency for teacher use

Procedure:

1. Cut scales A and B along lines indicated. Scale A represents trips taken (second addend). Scale B represents the starting point (first addend).

2. Taking scale A, line up the reference point (marked by the zero) directly above the first addend position on scale B (see illustration below). This signifies the starting point on your number line.



$4 + -7 = -3$
Place the zero of scale A above 4 on scale B.

3. Determine the distance and direction to move by looking at the second addend.

   For example: −7 means 7 spaces *left.*

   Follow this trip on scale A.

4. *Read your answer directly below this number on scale B. In the example, the answer would be* −3.

5. Use the slide rule illustrated to represent the following sums:

$4 + -1 =$	$4 + -2 =$
$4 + -4 =$	$4 + 3 =$
$4 + 2 =$	$4 + -3 =$

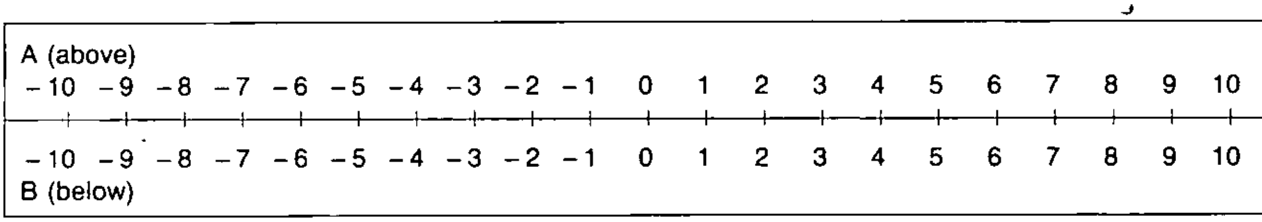
6. Have students try the following sums using the slide rule. Can any patterns or conclusions be drawn about the addition of integers using these examples?

(a) $4 + 5 =$	(d) $-3 + 7 =$	(g) $-7 + -2 =$
(b) $5 + 4 =$	(e) $-6 + 6 =$	(h) $-4 + -5 =$
(c) $3 + 4 =$	(f) $6 + -6 =$	

7. Ask students to do these questions in reverse.

For example: (c) rewrite as $-4 + 3 =$

Does this rewriting change the result?



Exercises for Students

1. Using your addition slide rule, find the following sums:

(a) $-8 + 4 =$	(d) $-6 + 7 =$	(g) $-7 + -2 =$
(b) $3 + -5 =$	(e) $6 + 2 =$	(h) $-6 + 6 =$
(c) $7 + -6 =$	(f) $-3 + -5 =$	

(i) Will you obtain a different result if you change the order of the numbers in questions (a) through (h)?

(j) What is the same in answers (c) and (d)?
What is different in answers (c) and (d)?

2. In your own words, state a rule to find the sum for:

(a) 2 positive numbers (question e above).
(b) 2 negative numbers (questions f and g above).
(c) 1 positive and 1 negative number (questions a, b, c, d, and h above).

3. Without using your scales, find the following sums:

(a) $-8 + -9 =$	(e) $-19 + -17 =$	(i) $-34 + -52 =$
(b) $3 + -2 =$	(f) $30 + 40 =$	(j) $34 + 52 =$
(c) $-7 + 4 =$	(g) $-42 + -41 =$	(k) $-60 + 60 =$
(d) $15 + -10 =$	(h) $-60 + 65 =$	(l) $87 + -87 =$

Compare your answers to (i) and (j). What do you notice about these 2 questions?

Compare your answers to (k) and (l). What do you notice about these 2 questions?

Subtraction of Integers

Level: Introductory Grade 8

Time: 1 class period (40–50 minutes)

Objective: To generate an understanding of the relationship between addition and subtraction of integers.

Prerequisite Skills: Directional meaning of positive and negative signs; experience with the addition slide rule

Procedure:

1. Using only *positive* numbers, have students do the following subtraction questions with the slide rule.

   (a) 4 − 2 = (c) 9 − 5 =
   (b) 8 − 3 = (d) 7 − 2 =

2. If the positive signs are understood for each number, what is the purpose of the subtraction sign in these questions?

   For example: 4 + −2 = 9 + −5 =
   8 + −3 = 7 + −2 =

   What similarities are there between the addition and subtraction questions? What differences are there?

3. Have students try these questions:

   (a) 3 − 5 = (c) 2 − 7 =
   (b) 5 − 7 = (d) 6 − 9 =

4. Ask students to write a related addition question for each example in *question 3*.

5. If any subtraction question can be written as addition, do we need the operation of subtraction?

6. Have students write the related addition question for each of the following:

   (a) 5 − −2 = (c) −9 − 5 =
   (b) 11 − −13 = (d) −7 − −6 =

7. Generate a rule for writing a subtraction question as addition.

Alternative Method: *The subtraction sign actually tells you to move in the opposite direction indicated by* the sign of the number.

For example: 8 − 5 means 8 − +5.

Hence, from starting point 8, move the opposite of 5 spaces right. Therefore, move 5 spaces left.

For example: −2 − 5 means −2 − +5. The starting point at +5 is understood. Move the opposite of 5 spaces right; therefore, move 5 spaces left.

For example: −7 − −5. From starting point −7, move the opposite of 5 spaces *left;* therefore, 5 spaces *right.*

Exercises for Students

1. Using your slide rule, complete the following subtraction problems:

   (a) $10 - 5 =$ (d) $0 - 7 =$ (g) $-2 - -4 =$
   (b) $2 - 4 =$ (e) $-5 - 5 =$ (h) $0 - -6 =$
   (c) $9 - 6 =$ (f) $-2 - 3 =$

2. Write the related addition question for each of the above questions and solve using your slide rule.

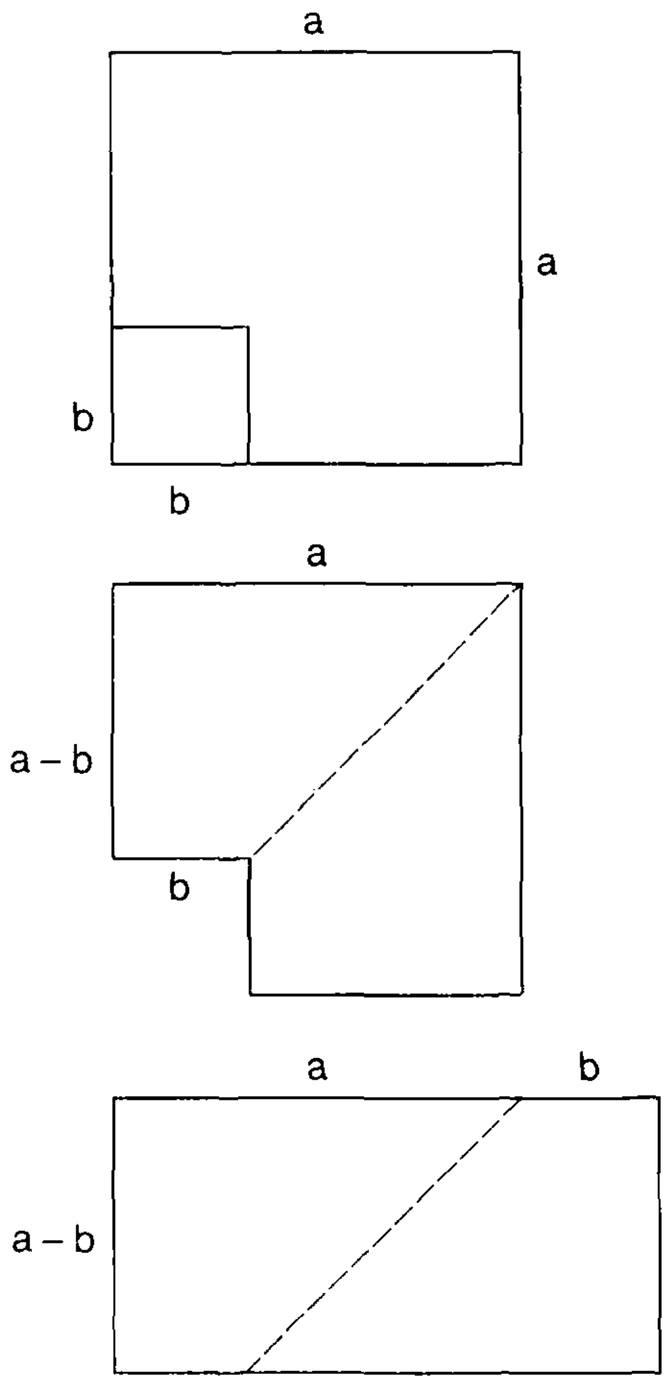
3. State a rule for subtracting 2 integers.

4. Solve the following problems without using the slide rule:

   (a) $-6 - 8 =$ (d) $-6 - -5 =$ (g) $-28 - -16 =$
   (b) $4 - 9 =$ (e) $5 - -7 =$ (h) $-19 - -19 =$
   (c) $0 - 7 =$ (f) $26 - 32 =$

39.

$$a^2 - b^2 = (a + b)(a - b)$$



40.

Polynomials

Level: Introductory Grade 9

Time: 1–2 class periods (40–80 minutes)

Objectives:

1. To identify characteristics of polynomials such as terms, coefficients, variables, and degrees.
2. To distinguish between polynomials of different degrees.
3. To distinguish among polynomials of the same degree.
4. To be able to give geometric analogs for parts of polynomials.

Prerequisite Skills: Knowledge of the concepts of linear, area, and volume measurement

Materials: Poster board (white, blue, orange, red, yellow, green, and black)

Preparation: The materials listed below are required for 1 set of polynomial models. All dimensions should be doubled if the set is intended for use as a teacher-demonstrator model. The dimensions can be any value as long as they are not integral multiples of 2 cm.

–25 2 cm × 2 cm squares (black—representing numbers)

10 each of the following:
–2 cm × 11.4 cm strips (red—representing x)
–2 cm × 14.4 cm strips (yellow—representing y)
–2 cm × 19 cm strips (green—representing z)

5 each of the following:
–11.4 cm × 11.4 cm squares (red—representing x^2)
–14.4 cm × 14.4 cm squares (yellow—representing y^2)
–19 cm × 19 cm squares (green—representing z^2)
–11.4 cm × 14.4 cm rectangles (white—representing xy)
–11.4 cm × 19 cm rectangles (blue—representing xz)
–14.4 cm × 19 cm rectangles (orange—representing yz)

3 each of the following:
–11.4 cm × 11.4 cm × 11.4 cm cubes (red—representing x^3)
–14.4 cm × 14.4 cm × 14.4 cm cubes (yellow—representing y^3)
–19 cm × 19 cm × 19 cm cubes (green—representing z^3)

1 each of the following rectangular prisms:
–11.4 cm × 11.4 cm × 14.4 cm (red, white—representing x^2y)
–11.4 cm × 11.4 cm × 19 cm (red, blue—representing x^2z)
–11.4 cm × 14.4 cm × 14.4 cm (yellow, white—representing xy^2)
–11.4 cm × 19 cm × 19 cm (green, blue—representing xz^2)
–11.4 cm × 14.4 cm × 19 cm (white, blue, orange—representing xyz)
–14.4 cm × 14.4 cm × 19 cm (yellow, orange—representing y^2z)
–14.4 cm × 19 cm × 19 cm (green, orange—representing yz^2)

NOTE: For the cubes and rectangular prisms, the squares and rectangles manufactured earlier can be duplicated, then taped together with invisible tape.

An interesting additional activity can be developed if the construction of the models is made into a problem-solving exercise.

Have the class try to determine the number of rectangles (2 dimensions) and the number of rectangular prisms (3 dimensions) that can be made by using just 3 basic measures.

Procedure:

1. Define the primary parts of the models for the class.

   (a) Black squares: 1 unit by 1 unit—value = 1.
   For example, 7 squares is said to be 7.

   (b) Red strip: 1 unit wide by x units long—value = x.
   For example, 8 red strips would be called 8x.

   (c) Yellow strip: 1 unit wide by y units long—value = y.
   For example, 3 yellow strips would be called 3y.

   (d) Green strip: 1 unit wide by z units long—value = z.
   For example, 12 green strips would be called 12z.

2. At this point, you may wish to have the class do questions 1 and 2 of the Exercises for Students.

3. Define terms and expressions using these models.

   (a) A term: one of the basic models.
   For example, 3 (3 black squares)
   5x (5 red strips)
   128z (128 green strips)
   (b) an expression: several basic models connected with plus signs.
   For example, 2x + 5 (2 red strips and 5 black squares)
   4x + 3y + 5z (4 red strips, 3 yellow strips, and 5 green strips)

4. At this point, you may wish to have the class do questions 3 and 4 of the Exercises for Students.

5. Expand the notion of terms and expressions to include the concept of 2 dimensions, that is, 2 variables. Locate all of the large squares and rectangles. Identify the lengths of the sides by matching strips.

   For example, students will discover that the white rectangle can be matched with a red strip on 1 side and a yellow strip on the other. Calling on the students' knowledge of area, point out that the area of the white rectangle can be found by using A = ab (altitude: red—x; base: yellow—y). Hence, the area of the white rectangle is xy. Similarly, the other squares and rectangles can be labeled according to their areas. Have the class identify all such areas.

   (a) red: red × red—x^2
   (b) yellow: yellow × yellow—y^2
   (c) green: green × green—z^2
   (d) blue: red × green—xz
   (e) orange: yellow × green—yz
   (f) white: red × yellow—xy

NOTE: Some students will find it helpful to record all their discoveries on a chart, which they can build as they go along.

6. Expand the notion of expressions to include the newly discovered 2-dimensional objects.
   For example, 2 orange rectangles—label 2yz.
   1 red square, 3 blue rectangles, 4 red strips, 5 black squares—label $x^2 + 3xz + 4x + 5$.

7. At this point, you may wish to have the class do questions 5 and 6 of the Exercises for Students.

8. Expand the notion of expressions to include the concept of 3 dimensions, that is, 3 variables. Locate all 3-dimensional objects in the set. Separate the cubes from the prisms. Recall that the value of such solids is calculated by multiplying length × width × height. Identify the various dimensions by matching strips, as was done with the rectangles.

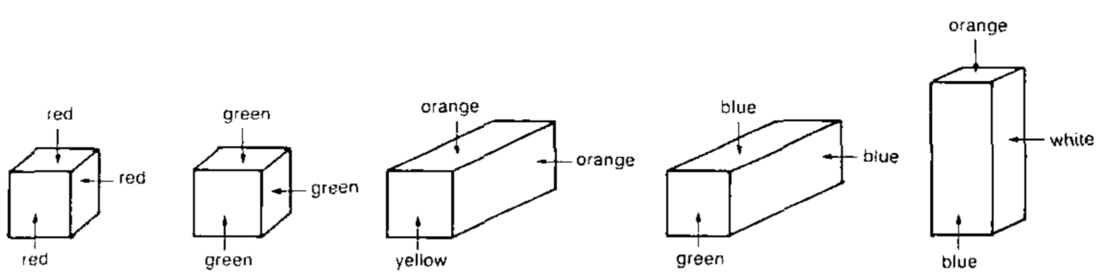
   For example, red cube length: red—x
   width: red—x
   height: red—x

   Hence, the volume of this cube is x^3. Similarly, all of the cubes and prisms can be labeled according to their volume. Have the class identify all such volumes.

   CUBES:
   (a) red cube: red × red × red—x^3
   (b) yellow cube: yellow × yellow × yellow—y^3
   (c) green cube: green × green × green—z^3

   PRISMS:
   (a) 2 red squares and 4 blue rectangles: red × red × green—x^2z
   (b) 2 red squares and 4 white rectangles: red × red × yellow—x^2y
   (c) 2 yellow squares and 4 white rectangles: yellow × yellow × red—y^2x (or xy^2)
   (d) 2 yellow squares and 4 orange rectangles: yellow × yellow × green—y^2z
   (e) 2 green squares and 4 orange rectangles: green × green × yellow—z^2y (or yz^2)
   (f) 2 green squares and 4 blue rectangles: green × green × red—z^2x (or xz^2)
   (g) 2 white rectangles, 2 blue rectangles, and 2 orange rectangles: red × yellow × green—xyz

9. Have the class practise labeling some of the 3-dimensional figures.



This would be labeled $x^3 + z^3 + y^2z + xz^2 + xyz$.

10. At this point, you may wish to have the class do questions 7 and 8 of the Exercises for Students.

Exercises for Students

1. Write the label (name) for each of the following:

   (a) 4 black squares
   (b) 7 red strips
   (c) 4 green strips
   (d) 6 yellow strips
   (e) 50 black squares
   (f) 1000 yellow strips
   (g) 91 red strips
   (h) 14 green strips

2. Show how to demonstrate the following by using the polynomial model set:

   (a) 6
   (b) x
   (c) 3y
   (d) 7z
   (e) 72
   (f) 31y
   (g) 1000x
   (h) 5z

3. Show how to demonstrate the following:

   (a) $3x + 5$
   (b) $2z + 4y$
   (c) $4y + 2z$
   (d) $3z + 1$
   (e) $x + y + z$
   (f) $2x + 2y + 2z$

   Are b and c the same?

4. Write the polynomials that are described by:

   (a) 6 black squares
   (b) 3 black squares and 2 red strips
   (c) 5 green strips and 4 yellow strips
   (d) 7 yellow strips, 2 green strips, and 1 red strip
   (e) 6 red strips, 3 yellow strips, and 9 black squares

5. Show how to demonstrate the following polynomials:

   (a) $2x^2 + 3y^2$
   (b) $2x^2 + 5x^2 + 6z^2$
   (c) $3xy + 4y^2 + 2x + 8$
   (d) $12x^2 + 3yz + 5y^2 + 4xz + 3y + 31$

6. Write the polynomials that are described by:

   (a) 3 red squares and 1 black square
   (b) 2 yellow squares and 6 green strips
   (c) 5 green squares, 3 blue rectangles, and 4 black squares
   (d) 2 red squares, 9 orange rectangles, and 5 yellow strips
   (e) 1 yellow square, 2 blue rectangles, 3 yellow rectangles, 4 green strips, and 5 black squares

7. *Work with a partner. Select a set of geometric objects.* Have your partner write the polynomial. Now, have your partner select the objects. You write the polynomial. Repeat this 4 times.

8. Find the objects described by the following polynomials:

   (a) y^3
   (b) xyz
   (c) x^2z
   (d) $2x^3 + 3xy$
   (e) $x^2y + 2xy^2 + 5$
   (f) $x^3 + 3x^2z + 4xy + 2z + 3$

Demonstrating the Degree of a Polynomial

1. When a polynomial *term* describes a 3-dimensional object, that term is said to be of *degree 3*. The following terms would all be of degree 3:

   (a) x^3 (red cube)
   (b) $2x^2y$ (prisms with red squares and white rectangles)
   (c) $4zy^2$ (prisms with yellow squares and orange rectangles)
   (d) $9xy^2$ (prisms with white, blue, and orange rectangles)

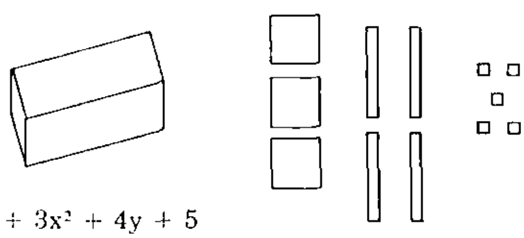
   If a term describes a 2-dimensional object, that term is said to be of degree 2. The following terms would all be of degree 2:

   (a) x^2 (red square)
   (b) $3xy$ (white rectangles)
   (c) $9z^2$ (green squares)

   If a term describes a 1-dimensional object, that term is said to be of degree 1. The following terms would all be of degree 1:

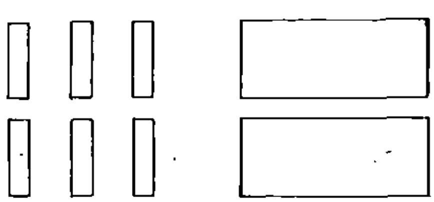
   (a) $3z$ (green strips)
   (b) x (red strip)
   (c) $9y$ (yellow strips)

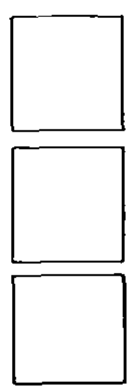
2. When a polynomial is arranged so that the term with the largest degree is written first, then the next largest and so on down, it is written in *descending order*. The following is an example of a polynomial written in descending order:

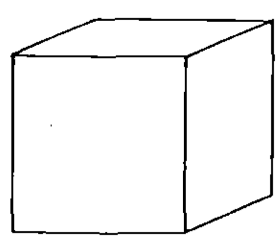


$$x^2z + 3x^2 + 4y + 5$$

3. When a polynomial is arranged so that the term with the smallest degree is written first, then the next larger and so on up, it is written in *ascending order*. The following is an example of a polynomial written in ascending order:







$$\underbrace{6 + 2xz}_{*} + 3y^2 + z^3$$

*If 2 objects have the same dimension, their order doesn't matter. The same is true of the degrees of the terms in the related polynomial.

4. The *coefficient* of a polynomial term tells how many objects there are. For example, $2y^3$ would be described by 2 yellow cubes. Because there are 2 cubes, the coefficient of $2y^3$ is 2.

Additional Exercises

It is recommended that the students work in pairs or groups of 3 to do the following exercises. One set of 3-dimensional models is required for each group.

1. Find the object set described by the term, then give its degree.

(a) $3z$	(d) $2xyx$	(g) $4y^3$
(b) $2x^2$	(e) $3yz$	(h) $4z^2$
(c) $4xy$	(f) $2z$	(i) $3y$

2. Give the degree of each *polynomial*. Check by picking out the object set.

(a) $15 + 7x + 2xy$	(e) $7y + 2z$
(b) $3xz + x^3$	(f) x^3
(c) $y^3 + y^2 + y + 1$	(g) $y^2 + x^2$
(d) $3 + 3z + 3z^2 + 3z^3$	(h) $3xyz + 2xy + 8x + 10$

3. Which of the following polynomial terms are of *degree 3* or describe 3-dimensional objects?

(a) x^3	(d) zy^3	(g) $7yx^2$
(b) x^2y	(e) $2y^3$	(h) 3
(c) $3x$	(f) $3x^2$	(i) xyz

4. Write the following polynomials in *descending* order. Check by ordering the related object set.

   (a) $7x + 3xz + 9 + y^3$
   (b) $3xyz + 8 + 5z + 3xy$
   (c) $1 + 2x + x^2$

5. Write the following polynomials in *ascending* order. Check by ordering the related object set.

   (a) $2x^3 + 3xy + x + 6$
   (b) $4x + 2y^3 + x^2z + 3$
   (c) $xz^2 + 2z^3 + 3yz$

6. Write the coefficient of each of the following terms.

(a) $2x^3$	(f) $7z$	(k) $5xy$
(b) $3y^3$	(g) $20x^3$	(l) $2xyz$
(c) $4x^2$	(h) $3y^2$	(m) $4yz$
(d) $0y^2$	(i) $2y^3$	(n) $2xy^2$
(e) $8z$	(j) $1001x$	(o) xz^2

7. Are $3y^2$ and $2y^3$ the same term? Find the 2 object sets.

   (a) Are they the same?
   (b) What does 3 mean in $3y^2$?
   (c) What does 3 mean in $2y^3$?

8. Pick out the objects needed to describe this polynomial: $2x^3 + 3y^2 + 2xy + z + 6$.

   (a) What is the degree of each term?
   (b) What is the degree of the polynomial?
   (c) Write the polynomial in ascending order.
   (d) Write the polynomial in descending order.
   (e) What is the coefficient of each term?

41.

Elastic Percent Approximator

Level: Introductory Grade 7

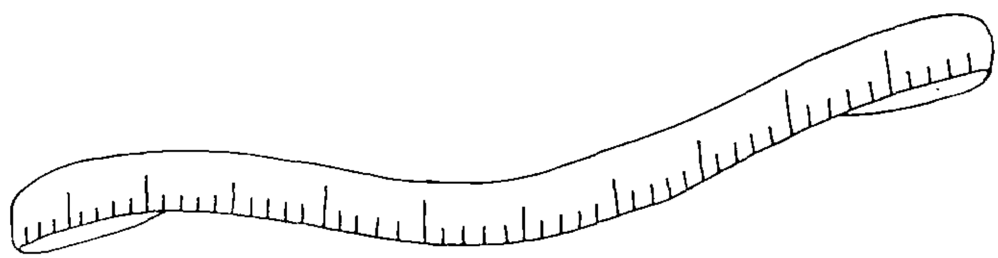
Time: 2 class periods (80–100 minutes)

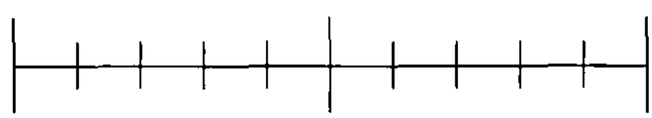
Objective: To approximate fractions as percentages in problem solving using geometric figures.

Materials and Preparation: A piece of elastic or a rubber band can be made into a percent calculator for approximating. You can use:

(a) a piece of elastic that is 8 cm long and 3 mm wide (the smaller the width, the more stretch the elastic has);

(b) a 6.5 to 8 cm piece of a rubber band that is 1 to 4 mm wide.





Two students work together to mark the elastic or rubber band. One stretches the material along the scale at the top, while the other marks the divisions. If the material is wide enough, the left end can be labeled 0%, the middle 50%, and the right end 100%. The labels indicate that the part of the elastic or rubber band with the marks is the reference set (100% quantity).

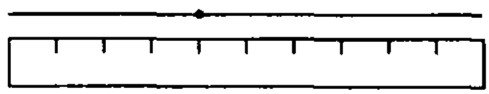
At this point, the students should experiment with the elastic or rubber band to see that the marks remain evenly spaced regardless of how much it is stretched. Students should be reminded that their answers will be approximate and that each segment represents 10% of the reference set because the reference set (100%) was divided into 10 equal parts.

The following section shows examples of student problems. Depending on your students, you may want to supply separate worksheets on the length, area, and volume concepts or include all 3 on the same worksheet. It is hoped that students will see that n% of a quadrilateral with opposite sides congruent can be shown in 2 ways and that n% of a 6-sided polyhedron with opposite faces congruent can be shown in 3 ways.

Exercises for Students

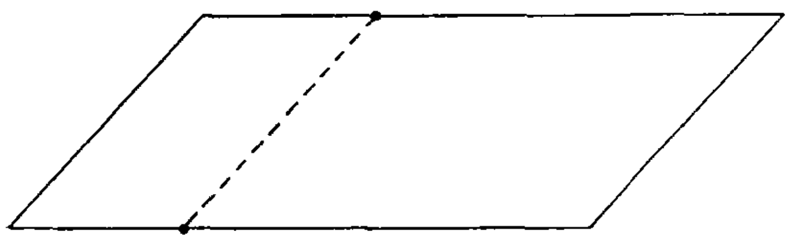
1. Divide this line segment so the left-hand part represents 40% of the entire line segment.

   (a) Place 0% on the left-hand endpoint.
   (b) Stretch the elastic until 100% falls on the right-hand endpoint.
   (c) Mark a point to represent 40% of the line segment.



2. Divide a parallelogram so the left-hand part represents 75% of the parallelogram.

   (a) 0% on left endpoint—100% on right endpoint.
   (b) Approximate 75% and mark.
   (c) Repeat on top segment.
   (d) Connect points.

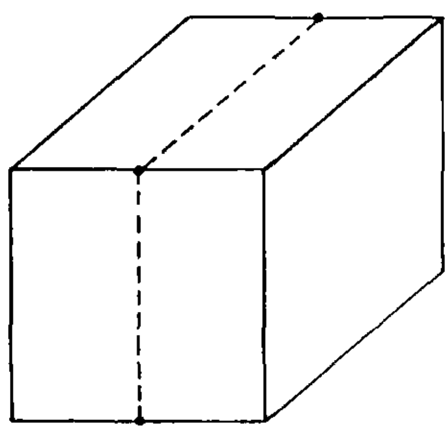


   NOTE: If the side dimension is less than the length of the elastic, the elastic could not be used to find 75% of the parallelogram.

3. Divide a cube so the left side shows 50% of the cube.

   (a) 0% on left endpoint—100% on right endpoint.
   (b) Mark 50%.
   (c) Repeat on other edges.
   (d) Connect points.

   NOTE: Similarly, the front 50% and the bottom 50% can be found.



These concepts could be developed into a series of lab activities using different lengths of string for the lines, transparent quadrilaterals and a felt-tip pen for marking, and transparent commercial polyhedron models with a felt-tip pen for marking.

42.

Fraction Circles

Level: Introductory Grade 7

Time: 1–2 class periods (40–80 minutes)

Objective: To add 2 fractional numbers.

Materials: Class set of fraction circles, at least 1 circle for each 2 students; 1 larger scale; tracer fraction circle

NOTE: A basic model for developing tracer circles is provided.

Procedure:

1. Empty the contents of the fraction circle envelope onto the desk.

2. Identify the fraction circle parts by placing them on the unit circle. On a piece of paper, have 1 of the partners prepare a chart that lists the color of each piece and its associated fraction.

3. To reinforce the notion of equivalent fractions, have the students find large pieces that can replace combinations of smaller pieces. One partner replaces, while the other records. (For example, one 1/2 piece could replace two 1/4 pieces or four 1/8 pieces or three 1/6 pieces, and so on. The second partner would write true statements such as 1/2 = 2/4. . . .

4. Have the students demonstrate and record some fraction addition questions and answers.

   (a) 1/4 + 1/4 = (c) 1/2 + 1/2 = (e) 3/8 + 4/8 =
   (b) 1/3 + 1/3 = (d) 2/4 + 1/4 = (f) 1/6 + 3/6 =

   NOTE: 1/4 + 1/4 is demonstrated by placing the first 1/4 piece at the zero mark and the second 1/4 piece adjacent to the first. The sum is read by proceeding from the zero mark clockwise to the end of the second piece.

5. Have the students replace each answer above with an equivalent fraction wherever possible.

6. Have the students write a rule that explains how to add fractions. Demonstrate the rule by preparing 5 examples. Record the examples. At this point, you may wish to have the class do questions from the Exercises for Students.

7. Have the students demonstrate the following to further verify their rule:

   (a) 1/2 + 1/4 = (c) 1/3 + 1/6 = (e) 1/4 + 1/6 =
   (b) 1/8 + 1/4 = (d) 1/3 + 1/4 =

8. Have the students demonstrate each of the above examples using only one kind of fraction piece. For example, 1/4 + 1/2 can be shown as 1/4 + 2/4 because 1/2 can

be replaced by two 1/4 pieces, as was previously shown. (At this point, the students should revise their adding fraction rules to accommodate all possible questions.)

9. Have the students prepare a chart to illustrate which fraction parts would be necessary to add combinations of given fractions:

addend 1	addend 2	possible answer (equivalent parts)
1/2	1/4	1/4
1/2	1/3	1/6

Have the students list all combinations of halves, thirds, quarters, sixths, and eighths.

10. Have the students examine their addition rule, revise it if necessary, then demonstrate its accuracy by writing 5 more examples.

NOTE: This activity can be adapted to subtraction of fractions.

Exercises for Students

1. List fractions equivalent to:

(a) 3/6
(b) 6/8
(c) 2/4
(d) 1/3
(e) 1/2
(f) 1/4

2. Use your fraction circles to add the following. Make sure that your answer is expressed as a basic fraction.

(a) 1/8 + 3/8 =
(b) 1/6 + 5/6 =
(c) 1/3 + 1/3 =
(d) 3/4 + 1/8 =
(e) 1/2 + 1/4 =
(f) 1/2 + 1/3 =
(g) 5/8 + 1/4 =
*(h) 1/4 + 1/3 =

*If this answer is not on your circle, what additional lines will you have to add to determine the correct answer?

3. A student has suggested that 1/2 + 1/4 = 2/6. Can this be correct? Use your fraction circle to demonstrate your answer.

4. Describe how you can determine what fraction parts will be needed to add any two fractions together. For example:

quarters + thirds: parts needed ____________
fifths + eighths: parts needed ____________

5. Use your fraction circles to show the following:

(a) 1/4
(b) 3/4
(c) 1 1/4
(d) Could you show 2 3/4? How many quarters be required?
(e) 1/8
(f) 5/8
(g) Could you show 4 1/8?
(h) How many parts would be required?

6. Use your fraction circles to add:

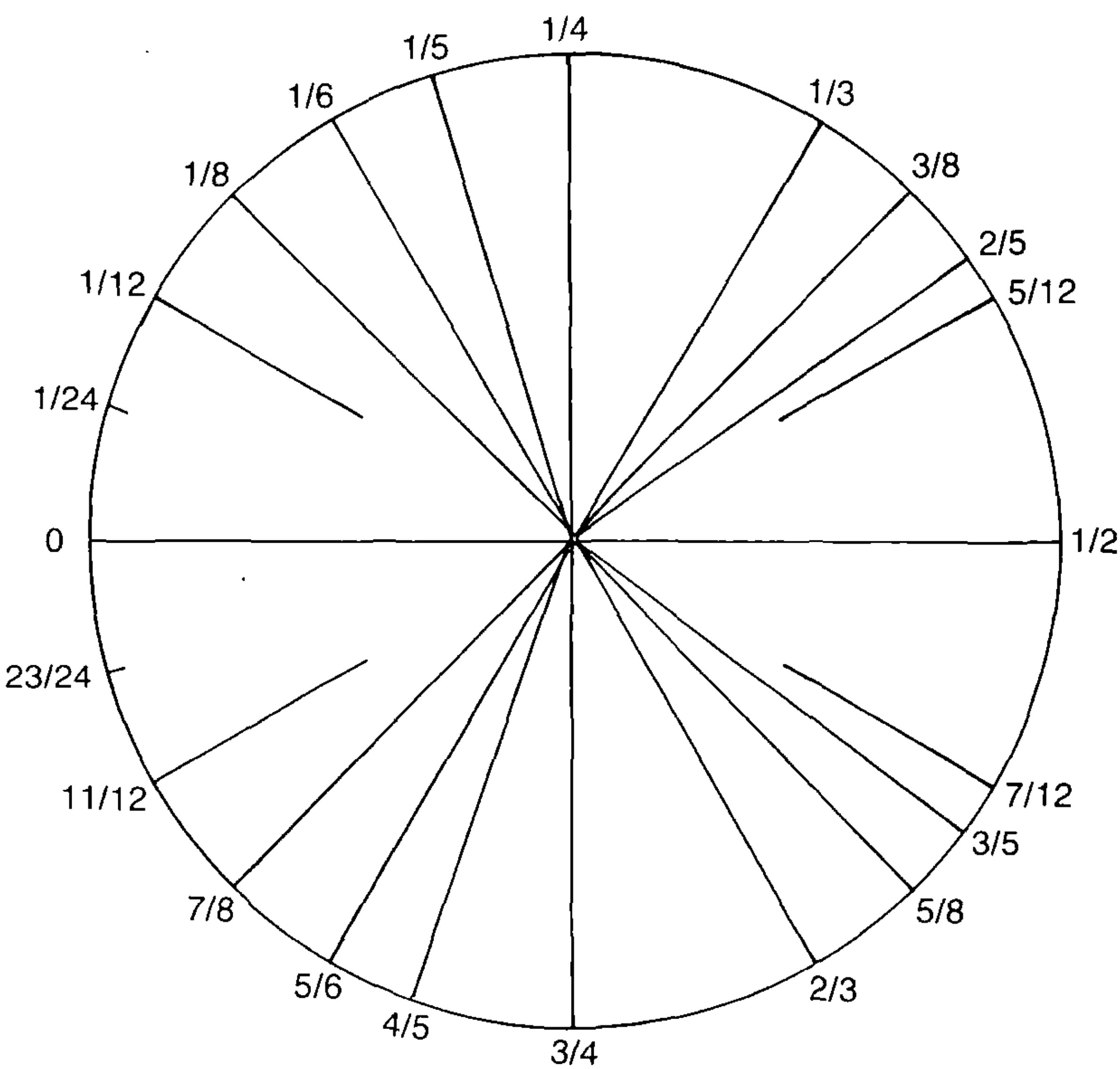
(a) 1/3 + 1 1/3 = (b) 3/4 + 2 1/4 = (c) 1 3/8 + 2 1/4 =

7. If you had fraction circle parts that included all fractions from 1/2 to 1/100 (1/2, 1/3, 1/4 . . . 1/99, 1/100), is there any pair of fractions for which you could not find the sum? If yes, list one such example, then tell what parts would be required to show this sum.

Preparation Notes

1. Copy the following fraction circle in sufficient quantity to allow one for each two students.

Fraction Circle Model



2. For consistency, it is suggested that the circles be duplicated according to the following color scheme:

1/2—green	1/4—yellow	1/8—tan
1/3—red	1/6—purple	1/12—orange

3. Duplicate the unit circle onto manila tag, if possible.

4. A larger copy, which could serve as a teacher demonstrator, could be prepared. The central angle for each fraction is listed below.

1/2 = 180°	1/6 = 60°	2/8 = 90°	1/12 = 30°	7/12 = 210°
1/3 = 120°	2/6 = 120°	3/8 = 135°	2/12 = 60°	8/12 = 240°
2/3 = 240°	3/6 = 180°	4/8 = 180°	3/12 = 90°	9/12 = 270°
1/4 = 90°	4/6 = 240°	5/8 = 225°	4/12 = 120°	10/12 = 300°
2/4 = 180°	5/6 = 300°	6/8 = 270°	5/12 = 150°	11/12 = 330°
3/4 = 270°	1/8 = 45°	7/8 = 315°	6/12 = 180°	

For the teacher demonstration model, attaching some felt to the backs of the circle parts and the centre of the unit circle helps to keep the parts from falling off.

For extended work, you may wish to build a secondary set containing thirds, fifths, fifteenths, and twentieths. For example:

1/5 = 72°
1/15 = 24°
1/20 = 18°

43.

O-oops!

Topic: Order of operations

Level: Grades 6–8

Number of Players: 3–4

Materials and Preparation: Set of O-oops cards, developed on 10 cm x 16 cm cards; a model of a possible set is provided here.

Procedure:

1. The dealer shuffles the cards and deals them all out. Each player matches the expression of numbers with the correct value, using the correct order of operations for the pairs in his or her hand. These pairs are placed face up on the table.
2. Play begins by each player's passing 3 of the remaining cards to the player to the right. If new pairs are formed from this action, each player adds these to the spread in front.
3. To begin the draw, the player to the left of the dealer draws a card from the hand of the player to his or her left. If the drawn card completes a pair, the player plays the pair face up with the others. Otherwise, the player keeps the card and the next player to the left draws from the player to his or her left.
4. Play continues until all the pairs are formed, leaving 1 player with the "O-oops" card. This person is the loser. Players drop out of the game when their cards are gone.

Variation: The player left with the "O-oops" card is the winner!

11	40	$12 \times 4 - 16 \div 2$	32	2
9	$40 \div 5 \times 4$	$7 - 3 - 2$	$4 + 3 + 2$	1000
3	$(3 + 12) \div 5$	5	17	1
$2 - 2 \div 2 + 2 \times 2$	$(3 + 2) \times 4 - 3$	$\frac{32 \div 8 + 3}{2 + 5}$	$8 \div 2 + 6 \div 2$	$5 + 1 \div 1$
$18 - 12 \div 2$	7	6	12	$(4 \times 5)^2$

O-oops 	21 21	$(5 + 2)(5 - 2)$ $(5 + 2)(5 - 2)$	4 4	18 18
31 31	$\frac{25 - 9}{5 - 1}$ $\frac{25 - 9}{5 - 1}$	$\frac{100 - 64}{10 - 8}$ $\frac{100 - 64}{10 - 8}$	$7 \times 4 + 3$ $7 \times 4 + 3$	$5 + 3 \times 2$ $5 + 3 \times 2$
49 49	$49 + 5 \times 0$ $49 + 5 \times 0$	8 8	10 10	0 0
$10 \div 5 \times 4$ $10 \div 5 \times 4$	$8 + 6 \times 1/3$ $8 + 6 \times 1/3$	$5 - 3 - 2$ $5 - 3 - 2$	$(12 - 2)^3$ $(12 - 2)^3$	400 400

44.

Turn a Card

Topic: Addition, subtraction, and multiplication of decimals

Level: Grades 5–6

Number of Players: 4

Materials: A deck of 48 cards, paper, pencil

Preparation: Cut 48 5 cm x 10 cm cards from tagboard. Write the following problems on the 48 cards:

8.2 + 26.3	30.4 − 2.9	0.28 × 134
7.62 + 5.2	31 − 3.8	7.2 × 4.62
3.78 + 83.64	48.6 − 24.9	1.48 × 4.2
17.63 + .986	34.23 − 17.6	0.008 × 3.7
0.002 + 1.01	23.02 − 5.3	40 × 0.072
12.7 + 428.6	572.3 − 130.9	36.3 × 43.1
73.06 + 53.007	16.7 − 2.008	0.423 × 101
87.3 + 45.7	116.3 − 39.6	16.3 × 3.3
57.6 + 65.73	204.37 − 92.66	35.9 × 1.8
3.26 + 4.72	42.8 − 16.7	3.68 × 2.4
14.68 + 25.734	24.26 − 0.08	5.39 × 0.62
36.22 + 3.8	0.426 − 0.299	46.2 × 0.3
328.16 + 39.28	261.7 − 48.9	3.8 × 7.6
53.76 + 39.28	4.64 − 1.08	4.72 × 5.5
78.6 + 43.67	72.003 − 24.64	2.5 × 10.7
128.48 + 39.72	43.006 − 4.841	4.62 × 2.3

Procedure: Shuffle and deal 12 cards to each player. Each player should put the cards in a stack, face down, in front.

Each of the 4 players will turn up the top card and solve the problem. Compare the 4 results. The player whose problem gives the largest number "wins" all 4 cards.

The "winning" player places the 4 cards at the bottom of the stack. Play continues for a predetermined time or until 1 player has all the cards. The person with the most cards is the winner.

45.

Integer Sums and Differences

Topic: Addition and subtraction of integers

Level: Grades 6–8

Number of Players: 2–4

Materials: 2 dice of different colors, a "Bingo" card for each player, markers

Procedure: Use 1 die to represent positive numbers and the second die to represent negative numbers.

Each player selects 1 of the 4 cards. On a player's turn, the dice are rolled, and the numbers are added or subtracted. The sum or difference is covered on each player's card. The first player to complete a row, column, or diagonal wins.

5	12	10	−1	−9
−10	−3	7	11	−8
−4	2	4	−6	−2
9	−7	−5	−11	1
8	6	3	0	−12

−9	−11	−1	−8	10
12	3	−10	−2	−6
8	−12	−7	−4	−5
2	6	11	7	1
−3	0	5	9	4

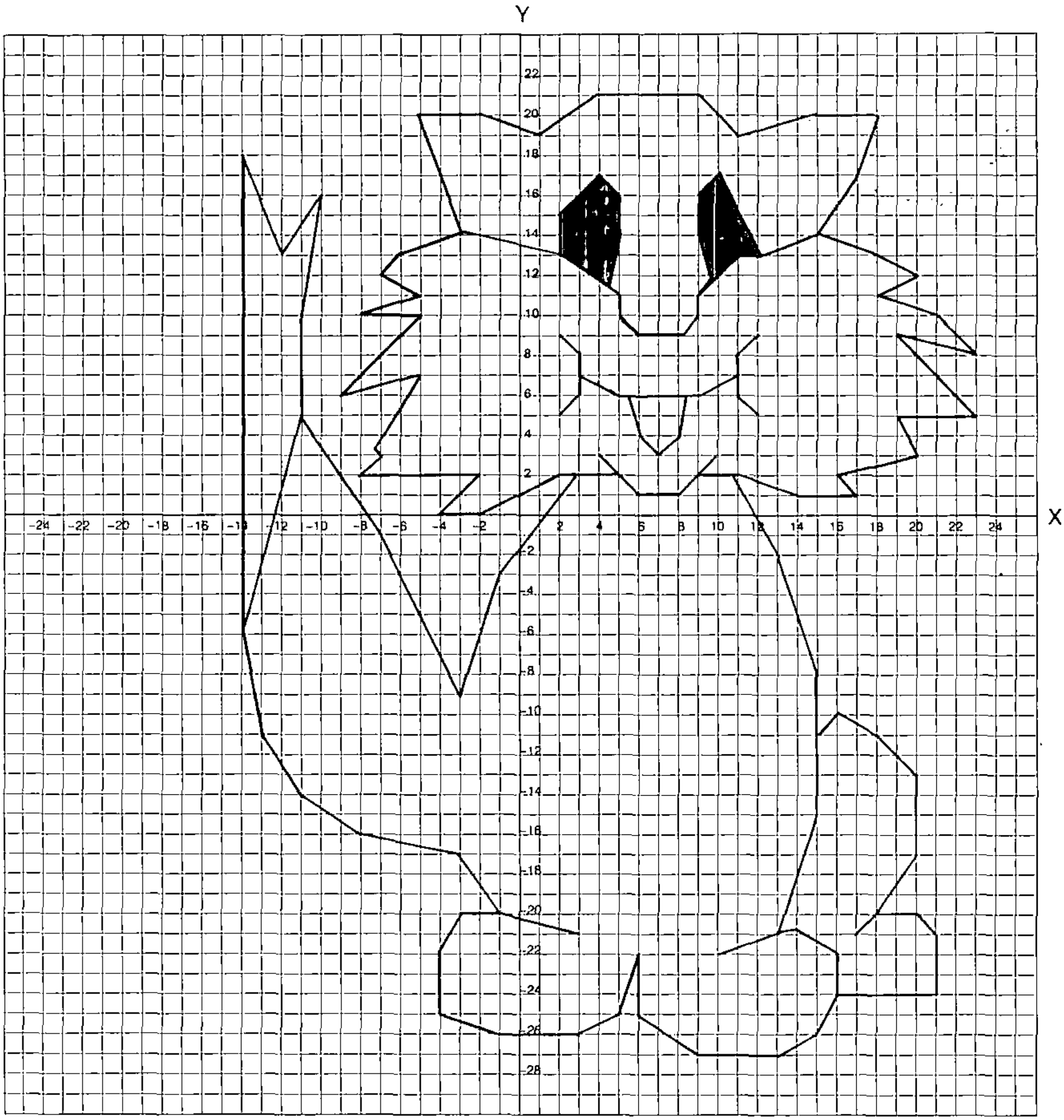
2	1	7	4	5
0	11	−3	9	6
8	−4	10	−12	−5
3	−2	−10	−9	−6
12	−7	−1	−8	−11

0	12	−11	6	−9
9	−7	8	5	−4
11	−3	−5	−1	3
7	−2	4	−10	2
−12	−6	−8	10	1

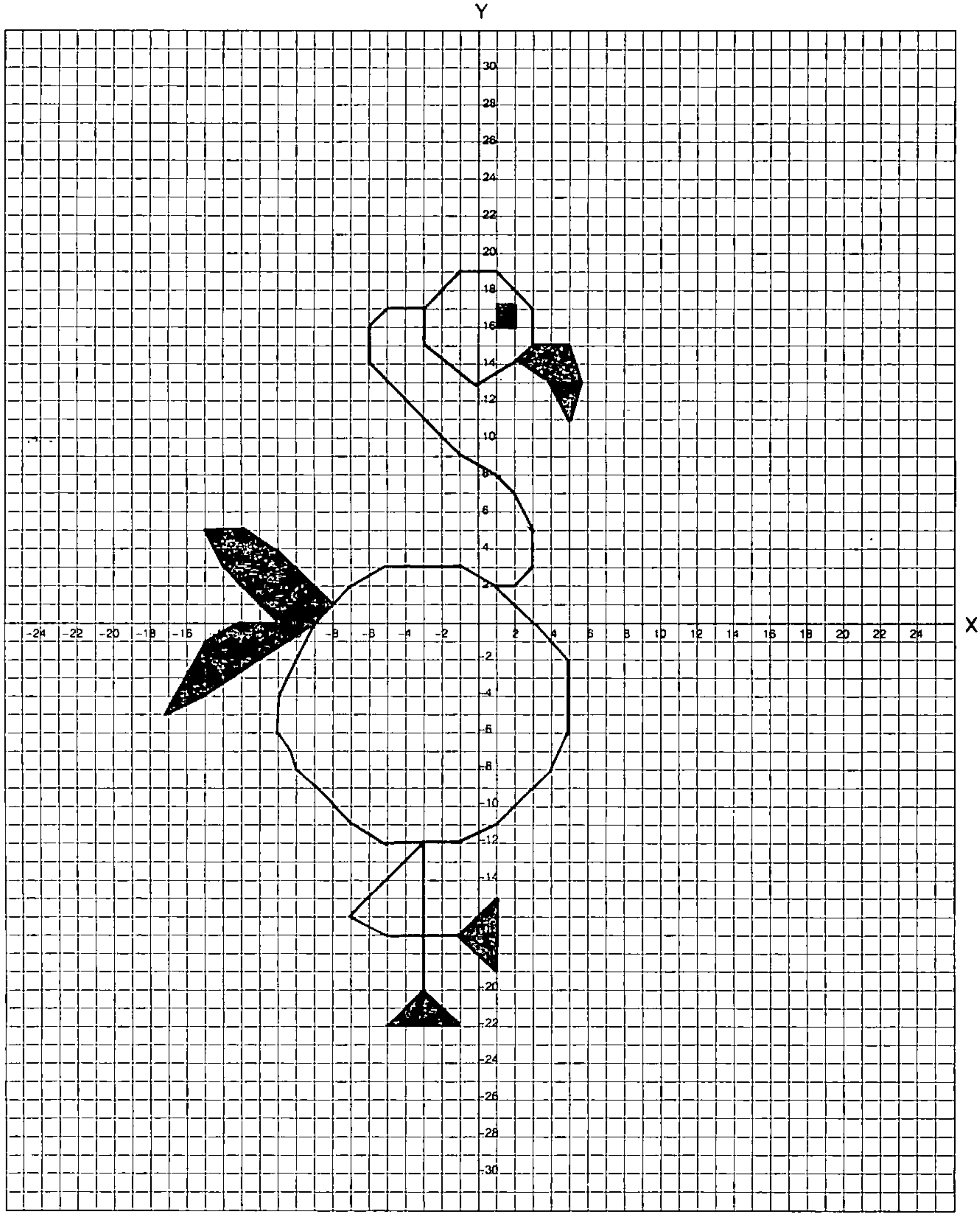
46.

Graphing

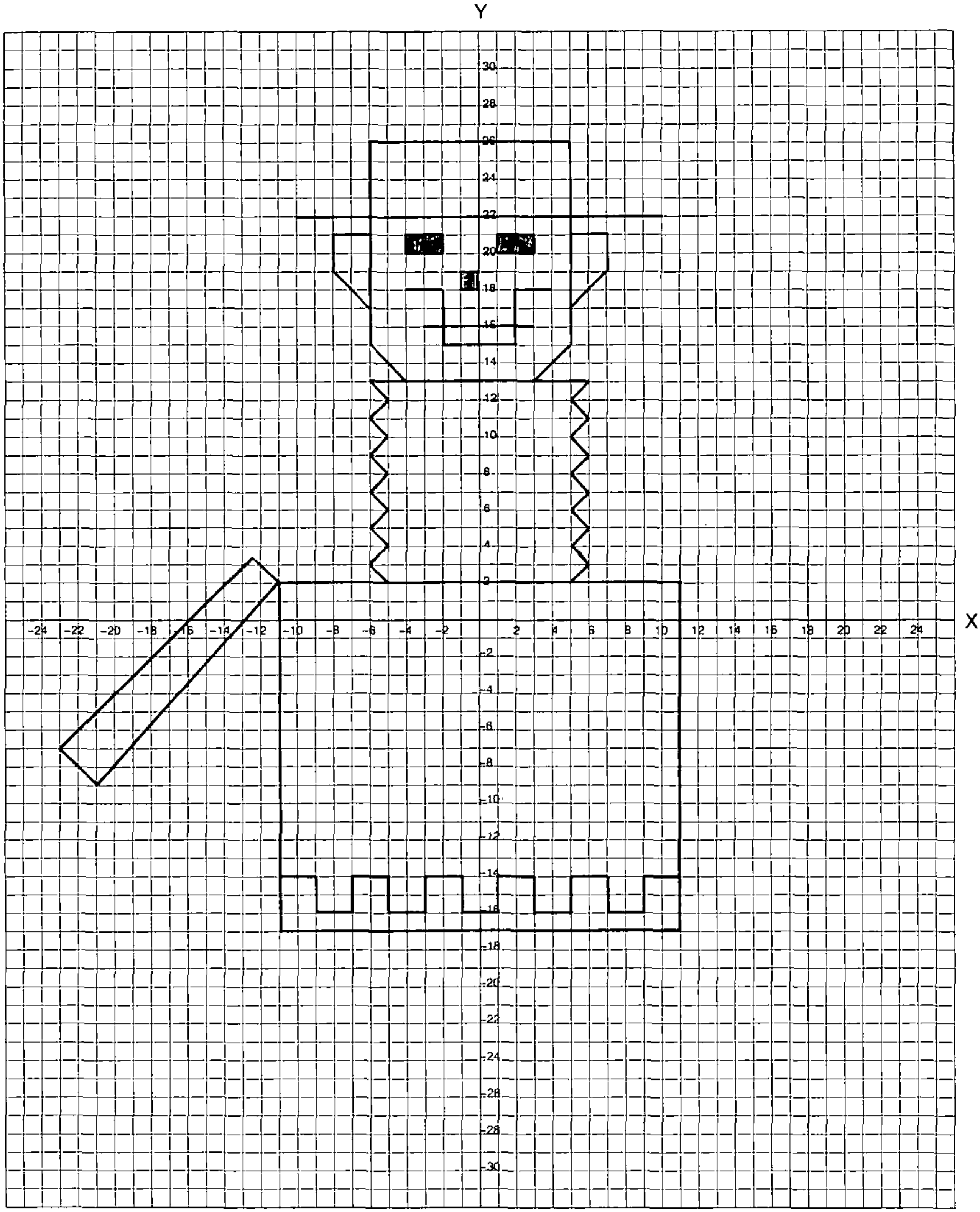
"Meow"



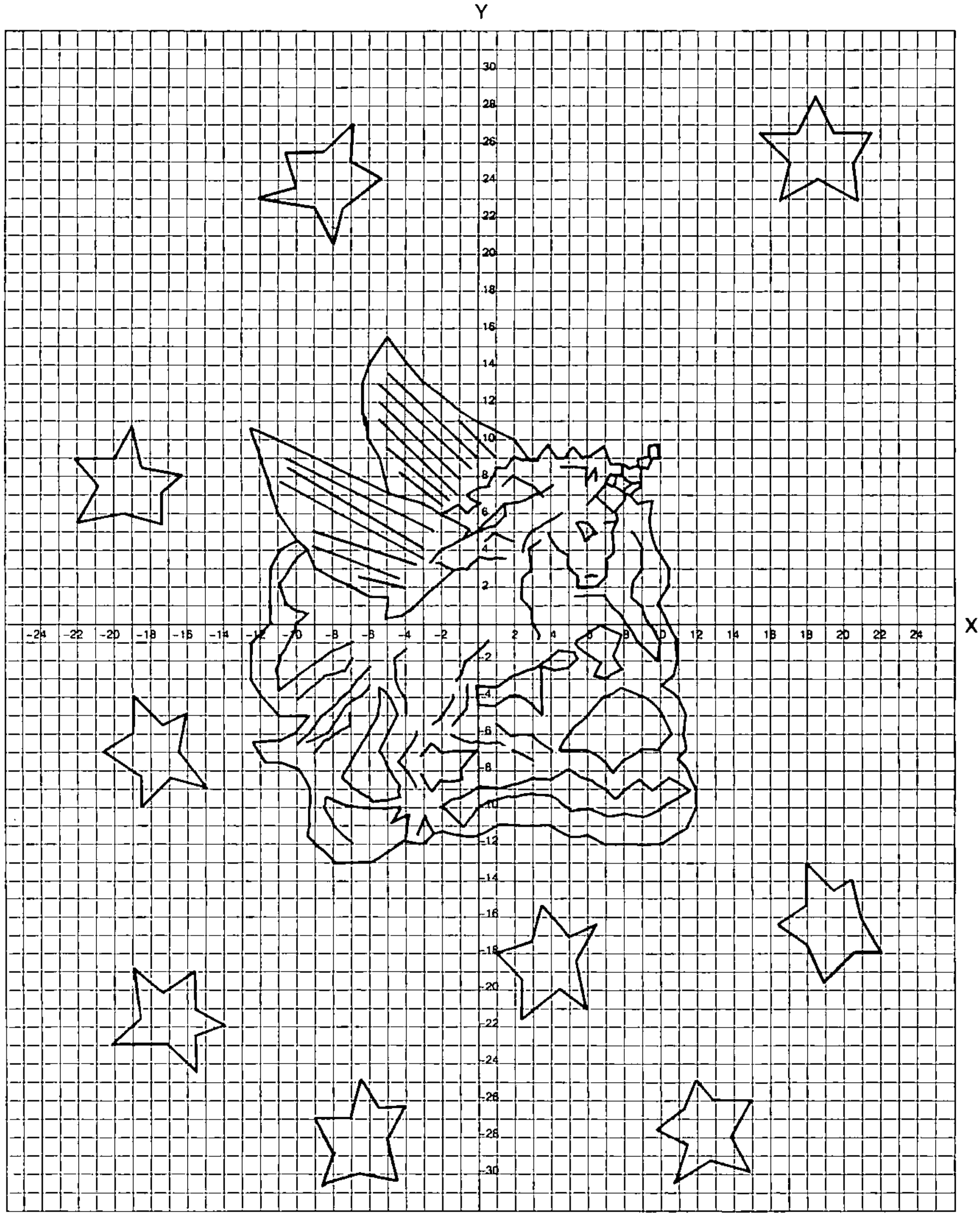
"Flamingo"



"Surprise"



"Unicorn"



GRAPHING KEY

"Meow"

(5, 2) (2, 2) (−2, 0) (−4, 0) (−2, 2) (−8, 2) (−7, 3) (−5, 7) (−9, 6) (−5, 10) (−8, 10) (−5, 11) (−7, 12) (−6, 13) (−3, 14) (−5, 20) (−2, 20) (1, 19) (4, 21) (9, 21) (11, 19) (15, 20) (18, 20) (17, 17) (15, 14) (18, 13) (20, 12) (18, 11) (21, 10) (23, 8) (19, 9) (23, 5) (19, 5) (20, 3) (16, 2) (17, 1) (14, 1) (11, 2) (9, 2)

(4, 3) (6, 1) (8, 1) (10, 3)

(−3, 14) (2, 13) (5, 11) (9, 11) (11, 13) (15, 14)

(5, 11) (5, 10) (6, 9) (8, 9) (9, 10) (9, 11)

(2, 13) (2, 15) (4, 17) (5, 16) (5, 14) (4 1/2, 11 1/2)

(9 1/2, 11 1/2) (9, 14) (9, 16) (10, 17) (11, 15) (12, 13)

(2, 9) (3, 8) (3, 6) (2, 5)

(12, 9) (11, 8) (11, 6) (12, 5)

(3, 7) (5, 6) (9, 6) (11, 7)

(7, 9) (7, 6)

(5 1/2, 6) (6, 4) (7, 3) (8, 4) (8 1/2, 6)

(3, 2) (−1, −3) (−3, −9) (−3, −17) (−1, −20) (3, −21) (−1, −20) (−3, −20) (−4, −22) (−4, −25) (−1, −26) (2, −26) (5, −25) (6, −22) (6, −16) (6, −25) (9, −27) (13, −27) (15, −26) (16, −24) (16, −22) (14, −20 1/2) (10, −22) (13, −21) (15, −15) (15, −8) (13, −2) (10, 2)

(15, −12) (16, −11) (18, −11) (20, −13) (20, −17) (18, −20) (17, −21)

(18, −20) (20, −20) (21, −21) (21, −23) (19, −24) (16, −24)

(−3, −12) (−7, −1) (−11, 5) (−12, 8) (−10, 16) (−12, 13) (−14, 18) (−14, −6) (−13, −11) (−11, −14) (−8, −16) (−3, −17)

"Flamingo"

(3, 0) (1, 2) (−1, 3) (−3, 3) (−5, 3) (−7, 2) (−8, 1) (−9, 0) (−10, −2) (−11, −4) (−11, −6) (−10, −8) (−9, −9) (−8, −10) (−7, −11) (−5, −12) (−3, −12) (−1, −12) (1, −11) (2, −10) (3, −9) (4, −8) (5, −6) (5, −2) (4, −1)

(2, 2) (3, 3) (3, 5) (2, 7) (1, 8) (−1, 9) (−2, 10) (−4, 12) (−6, 14) (−6, 16) (−5, 17) (−3, 17) (−3, 15) (−2, 14) (0, 13) (2, 14) (3, 15) (3, 17) (2, 18) (1, 19) (−1, 19) (−2, 18) (−3, 17)

(3, 17) (5, 15) (5 1/2, 13) (5, 11) (4, 13) (2, 14) SHADE

(1, 17) (2, 17) (2, 16) (1, 16) SHADE

(−8, 1) (−9, 2) (−11, 4) (−13, 5) (−15, 5) (−14, 3) (−13, 2) (−11, 0) (−9, 0) SHADE

(−11, 0) (−13, 0) (−15, −1) (−16, −3) (−17, −5) (−15, −4) (−14, −3) (−12, −2) (−10, −2) SHADE

(−3, 12) (−3, −14) (−3, −16) (−3, −18) (−3, −20) (−5, −22) (−1, −22) (−3, −20) SHADE

(−3, −12) (−5, −14) (−7, −16) (−5, −17) (−3, −17) (−1, −17) (1, −15) (1, −19) (−1, −17) SHADE

"Surprise"

(−11, 2) (11,2) (11, −17) (−11, −17) (−11, −14) (−9, −14) (−9, −16) (−7, −16) (−7, −14) (−5, −14) (−5, −16) (−3, −16) (−3, −14) (−1, −14) (−1, −16) (1, −16) (1, −14) (3, −14) (3, −16) (5, −16) (5, −14) (7, −14) (7, −16) (9, −16) (9, −14) (11, −14)

(−11, 2) (−12 1/2, 3 1/2) (−23, −7) (−21, −9) (−11, 2)

(−5, 2) (5, 2) (6, 3) (5, 4) (6, 5) (5, 6) (6, 7) (5, 8) (6, 9) (5, 10) (6, 11) (5, 12) (6, 13) (−6, 13) (−5, 12) (−6, 11) (−5, 10) (−6, 9) (−5, 8) (−6, 7) (−5, 6) (−6, 5) (−5, 4) (−6, 3) (−5, 2)

(−4, 13) (−6, 15) (−6, 26) (−5, 26) (5, 26) (5, 15) (3, 13)

(−10, 22) (10, 22)

(−6, 21) (−8, 21) (−8, 19) (−6, 17)

(5, 17) (7, 19) (7, 21) (5, 21)

(−4, 20) (−4, 21) (−2, 21) (−2, 20) (−3, 20) (−4, 21)

(1, 20) (1, 21) (3, 21) (3, 20) (2, 20) (1, 20)

(0, 18) (0, 19) (−1, 19) (−1, 18) (0, 18)

(−4, 18) (−3, 18) (−2, 18) (−3, 16) (−2, 15) (1, 15) (2, 15) (2, 16) (2, 18) (3, 18) (4, 18)

(−3, 16) (3, 16)

"Unicorn"

(−22, 9) (−20 1/2, 9) (−19, 10 1/2) (−18 1/2, 8 1/2) (−16 1/2, 8) (−17 1/2, 7) (−17 1/2, 5 1/2) (−19 1/2, 6) (−22, 5 1/2) (−21, 7 1/2) (−22, 9)

(−19, −4) (−17 1/2, −5 1/2) (−16, −5) (−16 1/2, −7) (−15, −9) (−17, −8 1/2) (−18 1/2, −10) (−18 1/2, −8 1/2) (−20 1/2, −7) (−19, −6) (−19, −4)

(−15 1/2, −19) (−15 1/2, −21) (−14, −22) (−15 1/2, −22 1/2) (−15 1/2, −24 1/2) (−17, −23) (−20, −23) (−18 1/2, −21 1/2) (−19, −19) (−15 1/2, −19)

(−6 1/2, −25) (−5 1/2, −26 1/2) (−4, −26 1/2) (−5, −28) (−4 1/2, −30 1/2) (−6 1/2, −30) (−8 1/2, −30 1/2) (−8, −29) (−9, −27) (−7, −27) (−6 1/2, −25)

(1, −18) (3, −17 1/2) (3 1/2, −15 1/2) (5, −17) (6 1/2, −16 1/2) (5 1/2, −18 1/2) (6, −21) (4 1/2, −20) (2 1/2, −21 1/2) (2 1/2, −19 1/2) (1, −18)

(12, −25) (13, −26) (15, −26) (14, −28) (15, −30) (13, −29 1/2) (11, −30 1/2) (11 1/2, −28 1/2) (10, −27 1/2) (11 1/2, −26 1/2) (12, −25)

(18, −13) (19 1/2, −14 1/2) (21 1/2, −14) (21, −16) (22, −18) (20 1/2, −18) (18 1/2, −19) (18, −17 1/2) (16 1/2, −16 1/2) (18, −15 1/2) (18, −13)

(15 1/2, 26 1/2) (17 1/2, 26 1/2) (18 1/2, 28 1/2) (19 1/2, 26 1/2) (21 1/2, 26 1/2) (20 1/2, 25) (21, 23 1/2) (18 1/2, 24) (16 1/2, 23) (17, 25) (15 1/2, 26 1/2)

(−12 1/2, 10 1/2) (−12, 9) (−11 1/2, 7 1/2) (−11, 6) (−10, 4 1/2) (−9 1/2, 4) (−9, 3) (−8, 2 1/2) (−7, 2) (−6,1 1/2) (−5 1/2, 1 1/2) (−5, 1) (−4 1/2, 1/2)

(−10, 4 1/2) (−11, 4) (−11 1/2, 3) (−11 1/2, 2) (−11, 1) (−11 1/2, 0) (−12 1/2, −1) (−12 1/2, −2) (−12 1/2, −3) (−12, −4) (−12 1/2, −4) (−13, −3 1/2) (−12 1/2, −4 1/2) (−11, −5) (−9 1/2, −5) (−10, −6) (−11, −6 1/2) (−11 1/2,−6 1/2) (−12, −6 1/2) (−12 1/2, −6 1/2) (−12, −7 1/2) (−11, −7 1/2) (−10, −8) (−9 1/2, −9) (−9 1/2, −10) (−9 1/2, −11) (−9 1/2-11 1/2) (−9, −12) (−8 1/2, −12 1/2) (−8, −13) (−7, −13) (−6, −13) (−5, −12 1/2) (−4, −12) (−3, −12) (−2 1/2, −11 1/2) (−2, −11 1/2) (−1, −11 1/2) (0-11 1/2) (1, −11) (2, −11) (3, −11) (4, −11) (5, −11 1/2 (6, −11 1/2) (7, −12) (8, −12) (9, −12) (10, −12) (11, −11 1/2) (11 1/2, −11) (12, −10) (12, −9) (11 1/2, −8) (11, −7 1/2) (11 1/2, −7) (11 1/2, −6) (11 1/2, −5) (11, −4) (10, −3 1/2) (10 1/2, −3) (11, −2) (11, −1) (10 1/2, 0) (10, 1) (10 1/2, 2) (10 1/2, 3) (10, 4) (9 1/2, 5) (9 1/2, 6) (9 1/2, 6 1/2) (9, 6 1/2) (8 1/2, 7)

(7, 5 1/2) (7 1/2, 5 1/2) (7 1/2, 6) (8, 6 1/2) (8, 7) (8 1/2, 7) (8 1/2, 7 1/2) (9, 7 1/2) (9, 8) (9, 8 1/2) (9 1/2, 8 1/2) (9 1/2, 9) (10, 9) (10, 9 1/2) (9 1/2, 9 1/2) (9, 9) (8 1/2, 9) (8 1/2, 8 1/2) (8, 8 1/2) (8, 8) (7 1/2, 8) (7 1/2, 7 1/2) (7, 7 1/2) (6 1/2, 7) (6, 6 1/2)

(9, 9) (9 1/2, 9) (9 1/2, 8 1/2)

(8, 8 1/2) (8 1/2, 8 1/2) (9, 8 1/2) (9, 8)

(7 1/2, 8) (8, 8) (8 1/2, 7 1/2)

(7, 7 1/2) (7 1/2, 7 1/2) (8, 7)
(6 1/2, 7) (7, 6 1/2) (7 1/2, 6)
(7 1/2, 8) (7, 8 1/2) (7, 9 1/2) (6, 9) (5 1/2, 9 1/2) (5, 9) (4 1/2, 9) (4, 9 1/2) (3 1/2, 9) (3, 9) (2 1/2, 9) (2, 9) (1 1/2, 8 1/2) (1, 8 1/2) (1/2, 7 1/2) (0, 7 1.2) (−1/2, 7) (0, 6 1/2) (−1/2, 6) (−1, 6 1/2) (−2, 6) (−1, 5 1/2) (−1/2, 5) (−1, 4 1/2) (−2, 4) (−2 1/2, 3 1/2) (−2, 3 1/2) (−1 1/2, 3) (−2 1/2, 2) (−3 1/2, 1) (−4 1/2, 1/2) (−5, −1/2) (−5, −1) (−6, −1) (−7, −1) (−8, −1 1/2) (−9, −2) (−10, −3) (−11, −3 1/2) (−11, −3 1/2) (−11, −3) (−11, −2) (−10 1/2, −1) (−10, 0) (−9 1/2, 1/2) (−10 1/2, 1) (−11, 2) (−10 1/2, 3) (−10, 3 1/2) (−9 1/2, 4)
(−12 1/2, 10 1/2) (−11, 10) (−10, 9 1/2) (−8, 8 1/2) (−7, 8) (−5, 7) (−3, 6 1/2) (−2, 6)
(−5, 7) (−5 1/2, 9) (−6, 10) (−6 1/2, 11 1/2) (−6 1/2, 13) (−6, 14) (−5, 15 1/2) (−4, 14) (−3, 13) (−1, 11 1/2) (0, 11) (1, 10 1/2) (2, 10) (3, 9)
(−5, 13 1/2) (0, 9)
(−5 1/2, 13) (−1/2, 8 1/2)
(−5 1/2, 12) (−1, 8 1/2)
(−5 1/2, 11) (−1, 8)
(−5, 9 1/2) (−1 1/2, 7 1/2) (−2 1/2, 4 1/2)
(−5, 9) (−1 1/2, 7)
(−4 1/2, 8) (−2, 6 1/2)
(−1, 11) (1, 9)
(−10 1/2, 9) (−2 1/2, 5)
(−10 1/2, 7 1/2) (−3, 4)
(−10 1/2, 6 1/2) (−3, 3 1/2)
(−9, 5) (−3 1/2, 3)
(−9, 4) (−4, 2 1/2)
(−6 1/2, 2 1/2) (−4, 2)
(−2, −10) (−1, −9 1/2) (0, −9) (1, −9) (2, −8 1/2) (3, 8 1/2) (4, −8 1/2) (5, −8) (6, −8 1/2) (7, −9) (7 1/2, −9 1/2) (8, −9) (9, −8 1/2) (10, −9) (10 1/2, −9 1/2) (10, −10) (9, −10 1/2) (8, −10 1/2) (7, −10 1/2) (6, −10) (5, −10) (4, −9 1/2) (3, −9 1/2) (2, −9 1/2) (1, −9 1/2) (0, −10) (−1, 11) (−2, −10)

(4 1/2, −7) (5, −6) (6, −5) (7, −4) (8,−3 1/2) (9, −4) (10, −5) (10 1/2, −6) (10, −7) (9, −7) (8, −7 1/2) (7 1/2, −8) (7, −7 1/2) (6, −7) (4 1/2, −7)

(7 1/2, 5) (7 1/2, 4 1/2) (7 1/2, 4) (7, 3 1/2) (7, 2 1/2) (6 1/2, 2) (5 1/2, 2) (5 1/2, 2 1/2) (5, 3) (5, 3 1/2) (4 1/2, 4) (4, 5)

(6, 4 1/2) (6 1/2, 5) (6, 5 1/2) (5 1/2, 5 1/2) (6, 4 1/2)

(8 1/2, 5) (9, 4) (9, 3) (8 1/2, 2) (8 1/2, 1 1/2) (9, 1) (9 1/2, 0) (10, −1) (10, −2) (9, −1) (8, 1/2) (7 1/2, 1) (7, 1 1/2) (6, 1 1/2) (5 1/2, 1 1/1)

(6, −1/2) (6 1/2, −1) (7, −2) (6 1/2, −3) (8, −2 1/2) (7 1/2, −2) (7 1/2, −1) (8, −1/2) (7, 0) (6, −1/2)

(5 1/2, −1) (6, −2) (7, −3)

(0, −3 1/2) (0, −4 1/2) (1, −4 1/2) (2, −4) (3, −4 1/2) (3 1/2, −5) (3 1/2, −4) (3 1/2, −3) (3 1/2, −2 1/2) (4, −2 1/2) (5, −2 1/2) (5 1/2, −2) (5, −1 1/2) (4 1/2, −1 1/2) (4, −2) (3, −2 1/2) (2, −3) (1, −3 1/2) (0, −3 1/2)

(−4, −1 1/2) (−4 1/2, −2) (−4 1/2, −3) (−4, −4) (−4, −5)

(−7, −2) (−7 1/2, −2 1/2) (−8 1/2, −3) (−9, −3 1/2) (−10, −4)

(−6, −2 1/2) (−6 1/2, −3) (−7, −3 1/2) (−7 1/2, −4) (−8, −4 1/2) (−8 1/2, −5) (−9, −5 1/2) (−10, −6 1/2)

(−7, −4) (−7 1/2, −4 1/2) (−8, −5 1/2) (−8 1/2, −6) (−9 1/2, −6 1/2)

(−6, −3 1/2) (−6 1/2, −4) (−7, −5) (−7, −5 1/2) (−8, −6) (−9, −7)

(−5 1/2, −3 1/2) (−5, −4) (−4 1/2, −5) (−5, −6) (−5 1/2, −7) (−5, −8) (−4 1/2, −9) (−4 1/2, −9 1/2) (−5, −9 1/2) (−6, −9 1/2) (−7, −9) (−7 1/2, −8 1/2) (−8, −7 1/2) (−7, −7) (−6, −6) (−5 1/2, −5) (−5 1/2, −3 1/2)

(−3 1/2, −11 1/2) (−3, −10 1/2) (−2 1/2, −11 1/2)

(−8 1/2, −9 1/2) (−7, −10) (−6, −10) (−5, −10) (−4 1/2, −10) (−4 1/2, −10 1/2) (−4, −10 1/2) (−4, −11 1/2) (−4 1/2, −12) (−7, −12) (−8, −11) (−8 1/2, −10) (−8 1/2, −9 1/2)

(−3 1/2, −6) (−4, −7) (−4 1/2, −7 1/2) (−4, −8) (−3 1/2, −9)

(−2 1/2, −5 1/2) (−2, −5) (−1 1/2, −4)

(−2 1/2, −6 1/2) (−2, −7) (−1, −7) (0, −7) (1/2, −7 1/2) (−1, −8) (−2, −8 1/2) (−2 1/2, −9) (−3, −8) (−3, −7) (−2 1/2, −6 1/2)

(1/2, −1 1/2) (−1/2, −2) (−1, −3)

(−1/2, −3 1/2) (−1, −4 1/2) (−1 1/2, −5 1/2)

(−1 1/2, −6) (0, −6 1/2) (1, −6 1/2)

(1, −5 1/2) (2, −6) (3, −6) (4, −7)

(2, −7) (3, −7 1/2)

(3 1/2, 4 1/2) (3, 4) (3, 3) (2 1/2, 2 1/2) (2 1/2, 2) (2 1/2, 1 1/2) (3, 1) (3, 1/2) (3, 0) (3 1/2, −1/2)

(6, 2 1/2) (6 1/2, 2 1/2)

(6, 8 1/2) (6 1/2, 8 1/2) (6 1/2, 8)

(4 1/2, 8 1/2) (5 1/2, 8 1/2) (6, 7 1/2)

(1 1/2, 7 1/2) (2, 8) (3, 7 1/2) (3 1/2, 7) (4, 7 1/2)

(1, 6) (1 1/2, 6 1/2) (2, 6 1/2) (3, 7)

(0, 5) (1/2, 5 1/2) (1, 5 1/2) (1 1/2, 6)

(2 1/2, 4) (3, 5) (4, 5 1/2) (4 1/2, 6)

(1/2, 4 1/2) (1, 5) (2, 4 1/2)

(−1/2, 3) (0, 3 1/2) (1, 3 1/2) (1 1/2, 3)

(−7, 27) (−7, 25) (−5 1/2, 24) (−7 1/2, 22 1/2) (−8, 20 1/2) (−9, 22 1/2) (−12, 2 1/2) (−10, 23 1/2) (−10 1/2, 25 1/2) (−8 1/2, 25 1/2) (−7, 27)

47.

The Price Is Right

Level: Grade 7

Objective: To reinforce computation skills involving unit price (ratio and proportion).

Materials: Play money, bell, scrap paper, pencils

Procedure:

1. Students' desks are arranged in a circle around a small table with a bell on it. When students wish to give an answer, they run to the centre of the room and ring the bell.
2. The sample problem is given, and all students who correctly answer the question, "Which is cheaper and by how much?" are awarded $200.
3. Review the calculations necessary to calculate the unit price and the difference in price.
4. Explain prizes:
   - $1,000 for the first student who correctly calculates the difference in price before any unit prices have been given.
   - $500 for the first student who correctly calculates the difference in price after 1 unit price has been given.
   - $300 for the first student who correctly calculates a unit price.
   - $200 for all students who correctly calculate the difference in price after both unit prices have been given.
   - $100 penalty for any wrong answer.
5. The student with the most money at the end of the game is the winner.

Variations:

1. Have three-quarters of the desks in the centre and a few desks outside. Label these $1,000, $1,500, and $2,000. Once students have earned enough money, they move to desks that are progressively farther from the bell.
2. If a bell is not available, have students line up to give their answers.

Sample Problem:

6 apples for $2.40 ($0.40 $.01 cheaper)	8 oranges for $3.28 ($0.41)

Problems:

15 balls for $3.30 ($0.22)	9 badminton birds for $1.62 ($0.18 $.04 cheaper)
25 dolls for $814 ($32.56 $.31 cheaper)	17 wagons for $558.79 ($32.87)
25 books for $149.50 ($5.98)	36 dictionaries for $213.12 ($5.92 $.06 cheaper)
15 prs. socks for $48.60 ($3.24 $.03 cheaper)	19 prs. pantyhose for $62.13 ($3.27)
8 dresses for $458.32 ($57.29 $35.17 cheaper)	3 suits for $277.38 ($92.46)

9 purses for $438.39
($48.71)

15 wallets for $430.35
($28.69 $20.02 cheaper)

3 chairs for $775.47
($258.49)

8 stools for $1,427.12
($178.39 $80.10 cheaper)

6 televisions for $4,734.78
($789.13 $857.99 cheaper)

2 video recorders for $3,294.24
($1,647.12)

3 pictures for $137.67
($45.89 $5.20 cheaper)

4 frames for $204.36
($51.09)

14 prs. curtains for $1,093.82
($78.13 $8.34 cheaper)

4 rugs for $345.88
($86.47)

3 radios for $259.41
($86.47 $10.98 cheaper)

2 tape decks for $194.90
($97.45)

8 prs. jeans for $235.76
($29.47)

15 sweaters for $299.70
($19.98 $9.49 cheaper)

26 calculators for $220.22
($8.47)

15 notebooks for $46.80
($3.12 $5.35 cheaper)

6 stoves for $4,499.22
($749.87 $124.22 cheaper)

5 fridges for $4,370.45
($874.09)

13 lamps for $1,283.88
($98.76 $98.82 cheaper)

5 tables for $987.90
($197.58)

12 prs. earings for $55.08
($4.59)

23 bracelets for $45.31
($1.97 $2.62 cheaper)

3 prs. runners for $109.41
($36.47 $16.14 cheaper)

8 prs. dress shoes for $105.22
($52.61)

6 track suits for $164.52
($27.42 $21.22 cheaper)

8 prs. joggers for $389.12
($48.64)

14 shirts for $362.88
($25.92)

21 blouses for $455.28
($21.68 $4.24 cheaper)

3 blankets for $55.26
($18.42)

4 pillows for $51.88
($12.97 $5.45 cheaper)

3 potted plants for $49.41
($16.47)

5 hanging plants for $64.90
($12.98 $3.49 cheaper)

8 newspapers for $3.92
($0.49 $0.25 cheaper)

6 magazines for $4.44
($0.74)

13 towels for $226.98
($17.46)

21 facecloths for $113.82
($5.42 $12.04 cheaper)

13 light bulbs for $11.57
($0.89 $8.01 cheaper)

5 spotlights for $44.50
($8.90)

48.

Mathematics as a Language

Objectives: 1. To make mathematics terminology easier for students to learn and understand.

2. To relate mathematics to language and to recognize roots in new words.

Materials: A class set of dictionaries, reference books on the history of words (*optional*)

Procedure: Many of our mathematical words are strange and meaningless to junior high school students. If we can relate these words to their roots, we can help the students to better understand the words, to enrich their vocabulary, and to coordinate mathematics with language (and health, human sexuality, religion, and so on).

Some common words I focus on are: monopoly, digit, invert, reciprocate, commute.

Following are some interesting words (not all mathematical) that I use to stimulate students' interest in words:

equator—equal distance from the poles

pupil—means small; the pupil in the eye reflects small images; the pupil in school starts off small

decimate—when the Romans conquered a town, they established their power by killing every tenth male

gymnasium—from the Greek word *gymnos,* "to train naked"

trapezoid—Euclid used *trapezion,* meaning "little table"

Tory—from the Irish word *toruidhe,* meaning "robber"

September, October, November, December—the Roman calendar started in March

mortgage—dead pledge; the land is taken away and is dead to the mortgagor forever

quintessence—the ancients believed in 4 main elements; the "ether" was number 5, which Aristotle associated with the sun, the moon, and the stars

Be prepared to handle such words as:

hyperactive—too active

hyperbole—exaggeration (too much)

heterosexual—interested in the other sex

mononucleosis—one nucleus (Red blood cells should have no nucleus. If they have one, they don't transport oxygen as they should.)

bisexual—two sexes

bible—two books

transvestite—clothing "across" the sexes

You can research words of your choosing. A sample assignment follows.

Math Word Roots and Prefixes

1. Each of the roots or prefixes below has a meaning that is carried over into several other words.
   (a) List 3 nonmathematical words that use the prefixes below.
   (b) State how they are related to the meaning given.
   (c) Use each properly in a sentence.
   (d) From what language did they originate?

mono-	one	deci-	ten
bi-	two	circum-	around
tri-	three	dia-	across
quad-, quat-	four	peri-	around
quint-	five	equi-	equal
poly-	many or several	hetero-	other
sept-, hept-	seven	hyper-	too much
oct-	eight	inter-	between, among
deca-	ten	trans-	across

2. From the list below, relate each word (or its root) to mathematical and nonmathematical usage.

digit	lateral	identity	base
commute	invert	reciprocate	
associate	distribute	denominator	

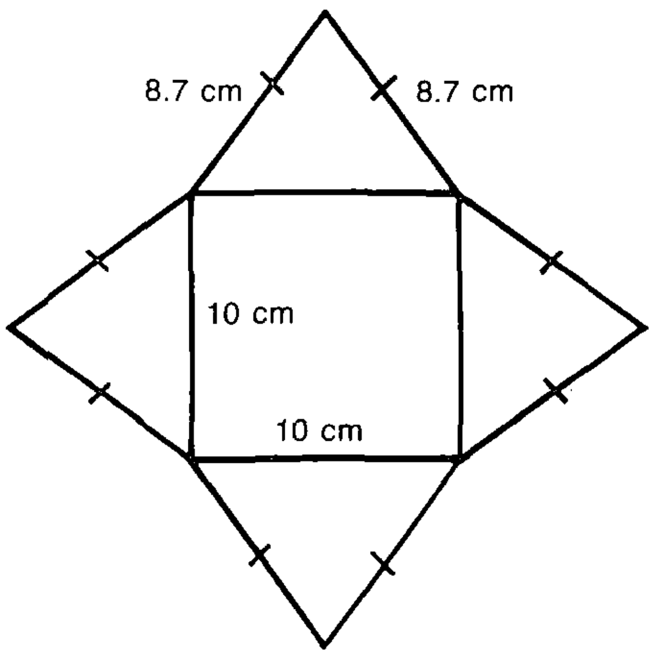
3. (a) Find the words below in the reference books suggested.
   (b) Briefly describe in *your own words* the history of 3 words from the following lists.
   (c) From the lists below, pick any 2 words that you find particularly interesting, and describe their history. (These words should be different from those of other students in your group.)

Thereby Hangs a Tale by C. Funk		*Word Origins and Their Romantic Stories* by W. Funk	
calculate	piano	algebra	intersect
camera	post	pupil	minus
caucus	salary	add	null
money	school	decadence	perimeter
gymnasium	trapezoid	decimate	quintessence
mortgage	Tory	decision	quart
one	volume	square	quarantine
pencil		hyper	subject
		integer	

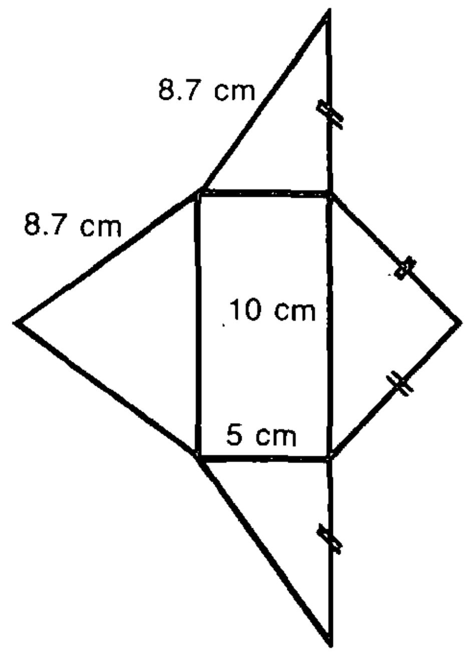
49.

Volume of a Pyramid

Use manila paper to make:



Make 1 of these.

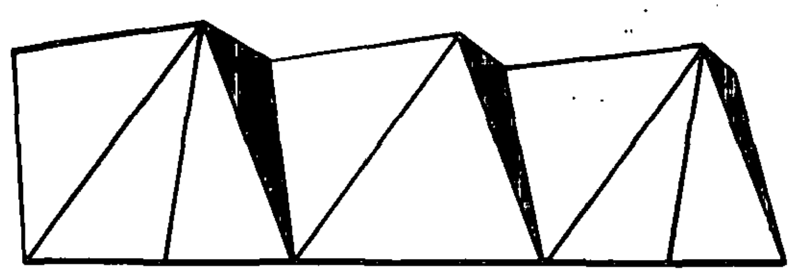


Make 4 of these.

Cut out, fold, and stick together to make 5 pyramids.

These 5 pyramids can be arranged so that 3 equal, identical pyramids are produced, and they can also be assembled to make a cuboid measuring *10* cm x *10* cm x *5* cm.

Suitably hinged with tape, the model will fold from 3 pyramids into a cuboid, and vice versa.

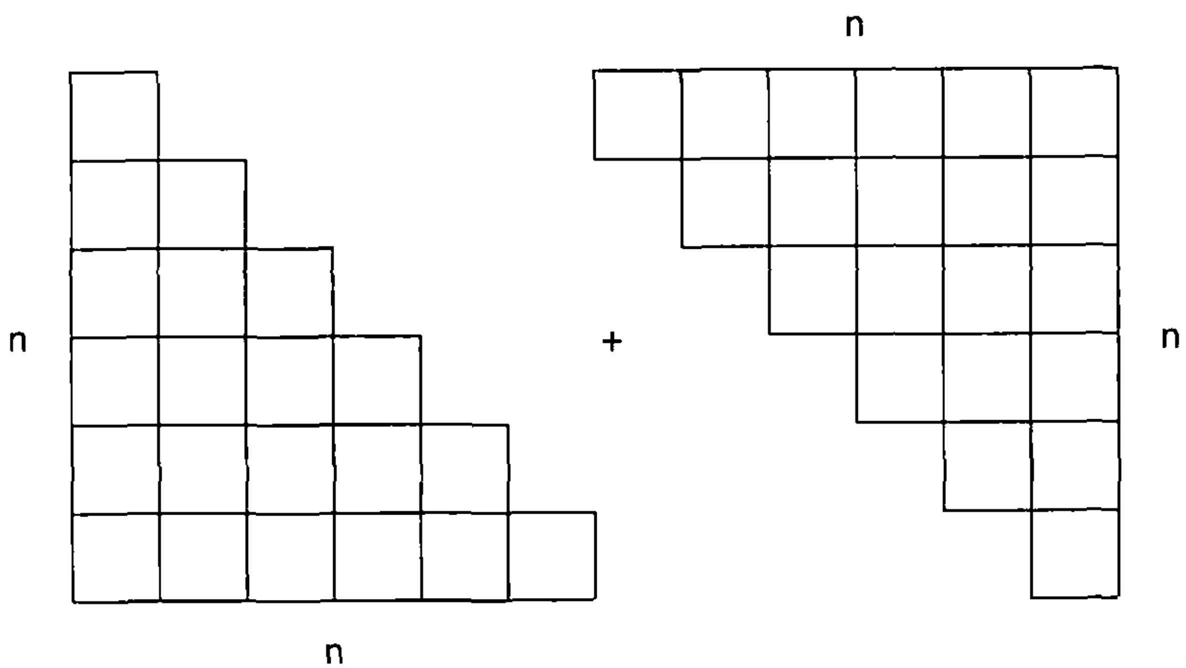


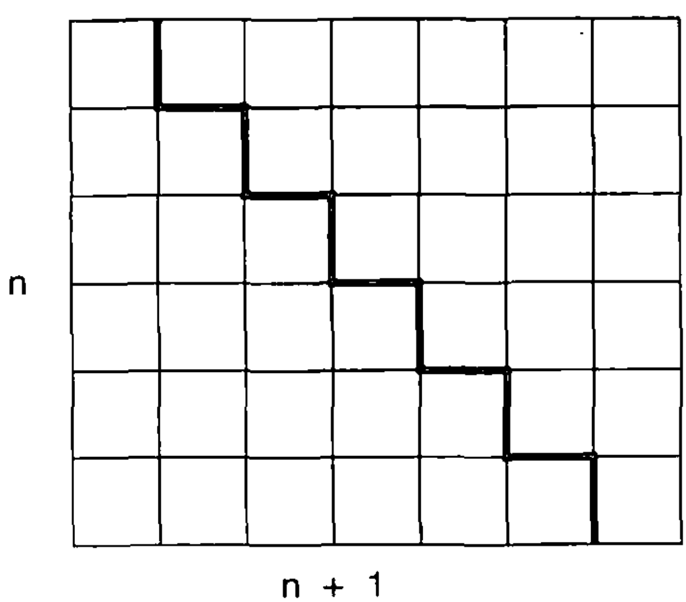
Since the 3 *pyramids make a cuboid*, you should be able to deduce the volume of 1 of the pyramids.

50.

Sum of First "n" Counting Numbers

The sum of the first n counting numbers = $1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$

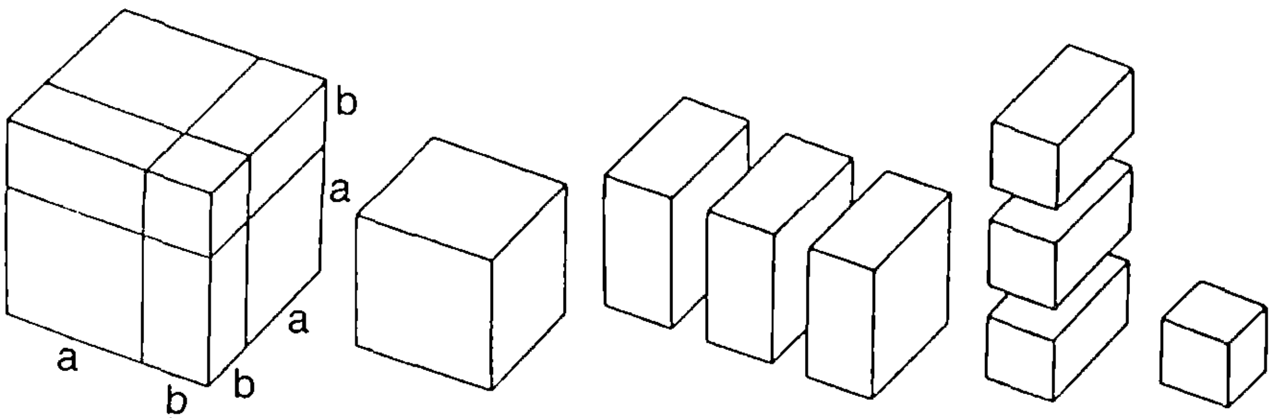




$$\frac{n(n+1)}{2}$$

51.

$(a + b)^3$



$(a + b)^3 =$ ________ + ________ + ________ + ________

52.

Advanced Mathematics Bingo

Topic: Review of mathematics concepts through trigonometry

Level: Grades 11–12

Number of Players: Entire class or small group

Materials: Regular set of BINGO cards and the Advanced Mathematics call cards; each player will need 25 beans for covers

Procedure:

1. One student or the teacher is the caller. The caller shuffles the call cards and begins by reading the problem on the top card.
2. The players work the problem and cover the answer on their BINGO card, if it appears there.
3. The first person to cover 1 row, 1 column, or 1 diagonal calls out "bingo." Play stops so that the caller may check the player's card to see if the correct numbers were covered. If so, then that player is declared the winner of that game. If not, play resumes until someone again calls out "bingo."

NOTE: the correct answers are given in the lower left-hand corner of the call cards.

Variations:

BINGO will consist of row 1 and the middle column, forming a "T."

BINGO will consist of column 1, column 5, and the middle row, forming an "H."

BINGO will consist of column 1 and row 5, forming an "L."

BINGO will be a blackout—all squares covered. Blackout can be played from the beginning or as a continuation of another type of BINGO. If this option is used, be sure that the students do not clear their cards after the first BINGO.

B	I	N	G	O
●	●	●	●	●
9	21	●	49	66
7	23	●	60	70
13	18	●	59	74
11	29	●	53	72

B	I	N	G	O
●	17	33	47	●
●	21	37	49	●
●	●	●	●	●
●	18	39	59	●
●	29	44	53	●

B	I	N	G	O
●	17	33	47	63
●	21	37	49	66
●	23	FREE	60	70
●	18	39	59	74
●	●	●	●	●

Sample Problems

If $321x = 321$, then $x =$ ___(1)___ .

A factor of every even number is ___(2)___ .

To solve equations with 3 unknowns, you usually have at least ___(3)___ equations.

$\sqrt{5^2 - 3^2} =$ ___(4)___ .

When 100 000 is written in scientific notation, the exponent on the 10 factor is ___(5)___ .

12_4 is what base 10 numeral? ___(6)___

The additive inverse of -7 is ___(7)___ .

An octagon has ___(8)___ sides.

The value of the determinant $\begin{vmatrix} 2 & 3 \\ 3 & 9 \end{vmatrix}$ is ___(9)___ .

Twice the number of sides in a pentagon is ___(10)___ .

What is the smallest prime number greater than 2^3? ___(11)___

$2\sqrt{36} =$ ___(12)___ .

A prime number greater than 11 and less than 17 is ___(13)___ .

The positive root of $x^2 - 13x - 14 = 0$ is ___(14)___ .

The value of $30 \cdot \sin 30°$ is ___(15)___ .

The value of $32 \cdot \cos 60°$ is ___(16)___ .

Five times the number of roots of a cubic equation *plus* the number of roots of a quadratic equation is ___(17)___ .

$6\sqrt[3]{27} =$ ___(18)___ .

$|-7| + |-12| =$ ___(19)___ .

The number of school days in a regular week *times* the number of seasons in the year is ___(20)___ .

The product of the roots of $x^2 - 10x + 21 = 0$ is ___(21)___ .

The sum of the roots of $x^2 + 22x + 40 = 0$ is ___(22)___ .

35_6 is what base 10 numeral? ___(23)___

The negative of the product of the roots of $2x^2 - 29x - 48 = 0$ is ___(24)___ .

$3^2 + 4^2 =$ ___(25)___ .

If $3x - 24 = 54$, then $x =$ ___(26)___ .

If $\frac{x}{3} = 9$, then $x =$ ___(27)___ .

The number of days in February 1982 is ___(28)___ .

A prime number greater than 24 and less than 30 is ___(29)___ .

The number of days in the month of June is ___(30)___ .

The number of days in the month of August is ___(31)___ .

The area of an isosceles right triangle with legs 8 cm is ___(32)___ .

$(\sqrt{9})(\sqrt{121}) =$ ___(33)___ .

One-half of 16 *plus* twice 13 is ___(34)___ .

Carol worked 4 problems in 20 minutes. At this rate, how many minutes will it take her to work 7 problems? ___(35)___

The area of a square with a diagonal of $6\sqrt{2}$ cm is ___(36)___ .

The number of days in April *plus* the number of days in a week is ___(37)___ .

Twice the prime between 17 and 23 is ___(38)___ .

The value of the determinant $\begin{vmatrix} 3 & -9 \\ 2 & 7 \end{vmatrix}$ is ___(39)___ .

If $\frac{x}{2} + x = 60$, then $x =$ ___(40)___ .

$3 \times 7 + 5 \times 4 =$ ___(41)___ .

How many days in 6 weeks? ___(42)___

If $19x = 817$, then $x =$ ___(43)___ .

$(4\sqrt{5})(\sqrt{5}) + 14 =$ ___(44)___ .

The product of the negative x– and y– intercepts of the ellipse

$\frac{x^2}{81} + \frac{y^2}{25} = 1$ is ___(45)___ .

If the slope of a line is $-1/46$, then the slope of a line perpendicular to it is ___(46)___ .

If $x + 2x = 141$, then $x =$ ___(47)___ .

$30 \tan 45° + 18 \cos 0° =$ ___(48)___ .

The area of a square with a perimeter of 28 is ___(49)___ .

The scale on a map is 1 cm = 20 m. How many metres are 2 1/2 cm on the map? ___(50)___

A prime number 1 more than $10\sqrt{/25}$ is ___(51)___ .

Thirteen times the ordinate of the vertex of $f(x) = 4 - x^2$ is ___(52)___ .

Find the tenth term in the sequence 17, 21, 25, 29, . . . ___(53)___

Find the geometric mean between 27 and 108. ___(54)___

Find the eleventh term in the sequence 5, 10, 15, 20, . . . ___(55)___

If $f(x) = x^2 + x$ and $g(x) = x + 3$, find $f(g(x))$. ___(56)___

If $f(x) = x^2 + 3x + 3$, find $f(6)$. ___(57)___

How many terms are in the sequence 18, 24 . . . 336? ___(58)___

f $\log_3 x = 3$, find $2x + 5$. ___(59)___

How many degrees in $\frac{\pi}{3}$ radians? ___(60)___

Find the arithmetic mean between 54 and 68. ___(61)___

If $f(x) = x^2 - 2$, find $f(8)$. ___(62)___

If $\log_4 x = 3$, find $x - 1$. ___(63)___

The absolute value of the product of the abscissas of the foci of

$\frac{x^2}{9} + \frac{y^2}{25} = 1$ is ___(64)___ .

If $\log_X 216 = 3$, find $5(2x + 1)$. ___(65)___

If $f(x) = x^3 + 2$, find $f(4)$. ___(66)___

If $\log_{16} x = \frac{3}{2}$, find $x + 3$. ___(67)___

The product of the positive values of the x– and y– intercepts of

$\frac{x^2}{289} + \frac{y^2}{16} = 1$ is ___(68)___ .

$\sqrt{49} \cdot \sqrt{100} - 1 =$ ___(69)___ .

The product of the coordinates of the centre and the length of the diameter of $x^2 + y^2 - 14x - 2y + 25 = 0$ is ___(70)___ .

The number of days it rained in a story about Noah *plus* the number of days in the month of March is ___(71)___ .

If $\log_3 x = 4$, find $x - 9$. ___(72)___

$3\sqrt{625} - 2 =$ ___(73)___ .

$0.0074 \times 10^4 =$ ___(74)___ .

If $\log_X 25 = 2$, find $3x^2$. ___(75)___

53.

Crazy Conic

Topic: Origin-centred conics—algebra 2 or analytic geometry

Level: Grades 11–12

Number of Players: 3–4

Materials: Set of "Crazy Conic" playing cards

Procedure:

1. The dealer shuffles the cards and deals them all out. Players match the graph of a conic with the corresponding equation for those pairs in their hand and put the pairs face up on the table.
2. Play begins by each player's passing 3 of the remaining cards to the player to his or her right. If new pairs are formed from this action, players add these to the spread in front of them.
3. To begin the draw, the player to the left of the dealer draws a card from the hand of the player to his or her left. If the drawn card completes a pair, the player plays the pair face up with the others. Otherwise, the player keeps the card, and the next person to the left draws from the player to his or her left.
4. Play continues until all the pairs are formed, leaving 1 player with the "Crazy Conic" card. This person is the loser. As players' hands are depleted, they drop out of the game.

Variation: The player left with the "Crazy Conic" card is the winner!

Equation Cards

Equation cards should display the following equations, 1 per card:

$x^2 + y^2 = 36$

$x^2 + y^2 = 25$

$(x - 4)^2 + (y - 4)^2 = 16$

$(x + 3)^2 + (y + 3)^2 = 9$

$(x - 4)^2 + (y - 6)^2 = 25$

$(x + 3)^2 + (y - 2)^2 = 25$

$y = x^2$

$y = \frac{1}{2}x^2$

$y = \pm\sqrt{x}$

$y = -x^2$

$y = x^2 - 3$

$y = (x + 3)^2$

$y = (x - 2)^2 - 4$

$\frac{x^2}{36} + \frac{y^2}{16} = 1$

$\frac{x^2}{36} + \frac{y^2}{4} = 1$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$

$\frac{x^2}{100} + \frac{y^2}{256} = 1$

$\frac{4x^2}{81} + \frac{y^2}{36} = 1$

$\frac{x^2}{36} + \frac{(y - 5)^2}{16} = 1$

$\frac{(x - 4)^2}{9} + \frac{(y - 3)^2}{25} = 1$

$\frac{x^2}{9} - \frac{y^2}{25} = 1$

$\frac{y^2}{16} - \frac{x^2}{9} = 1$

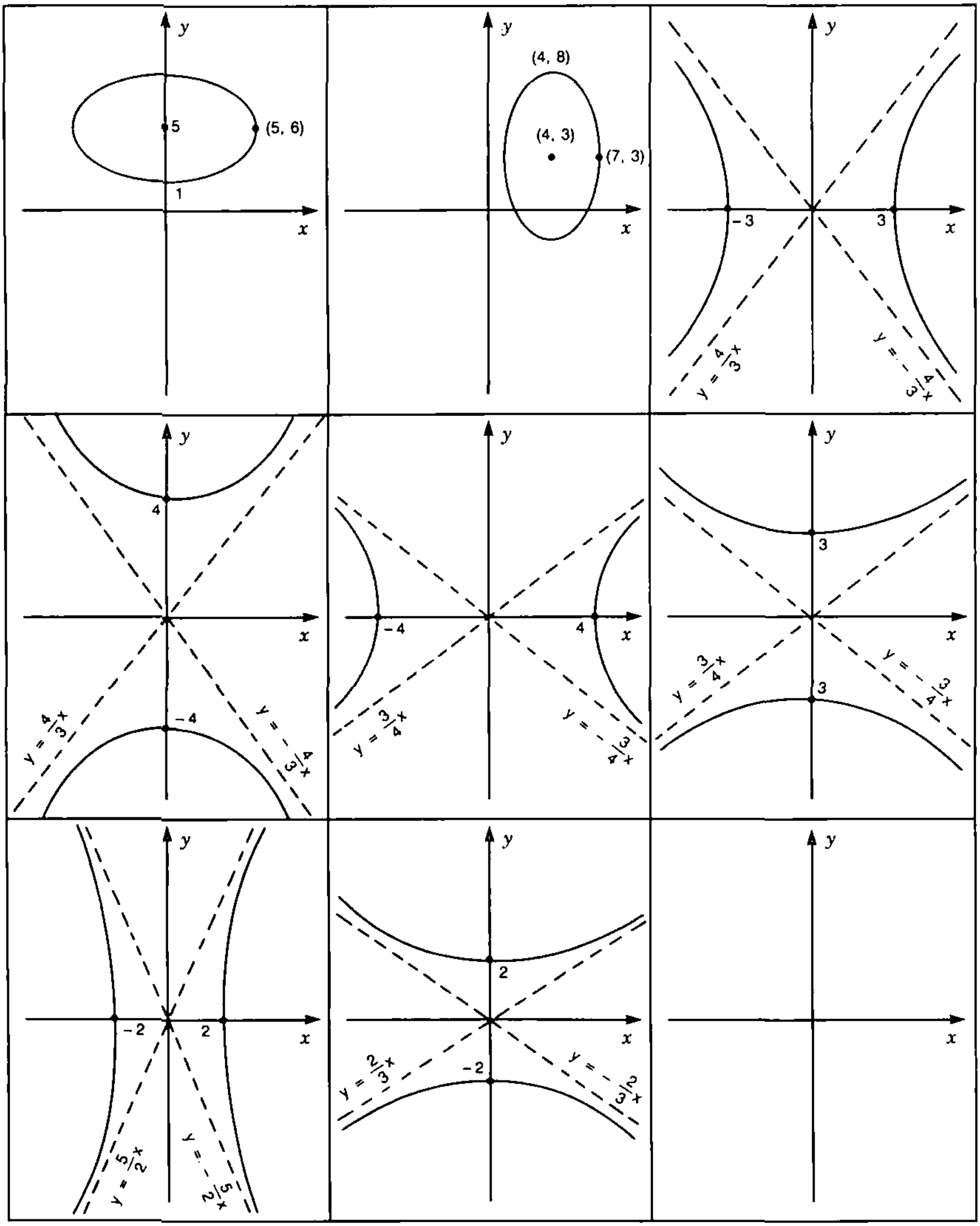
$\frac{x^2}{16} - \frac{y^2}{9} = 1$

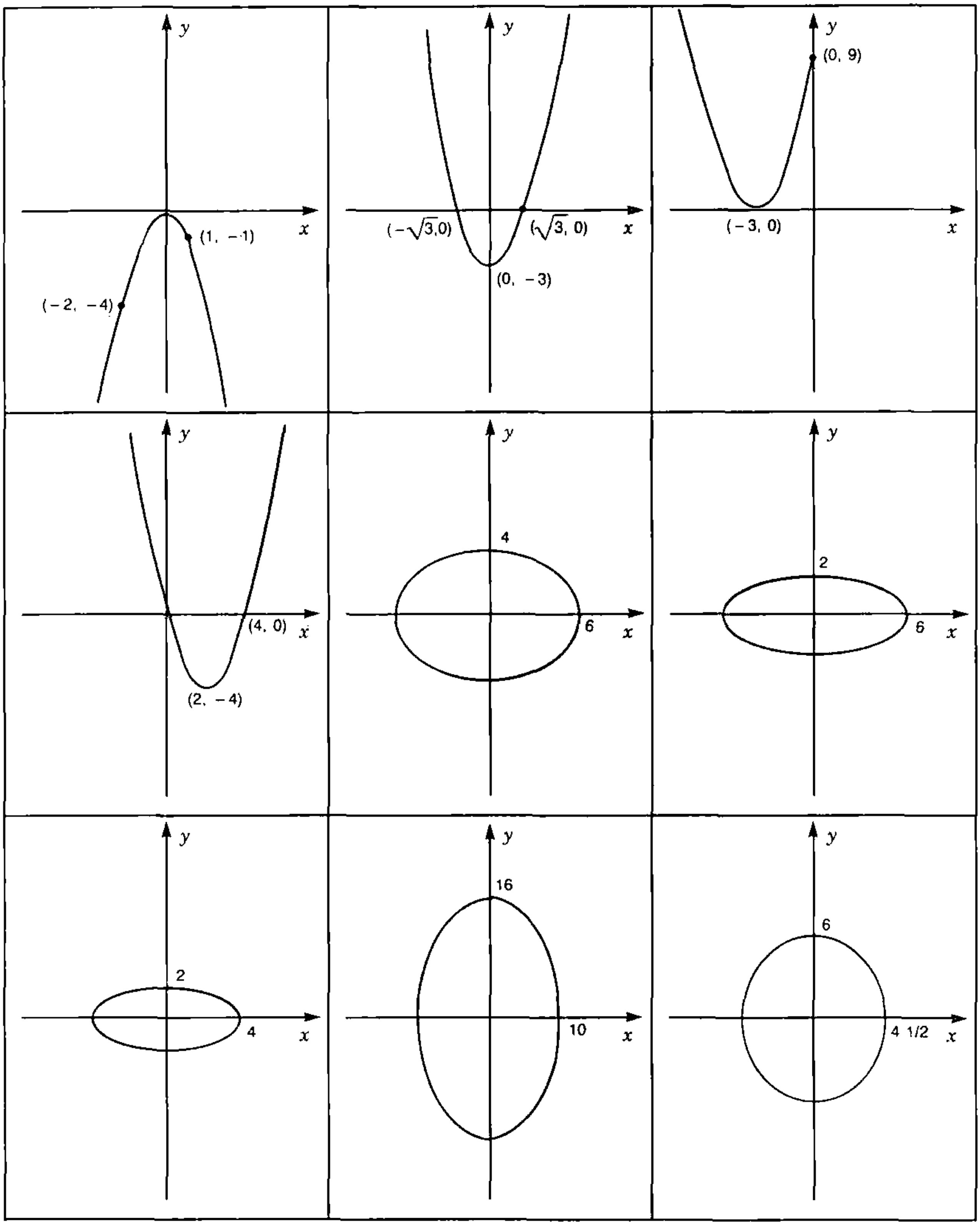
$\frac{y^2}{9} - \frac{x^2}{16} = 1$

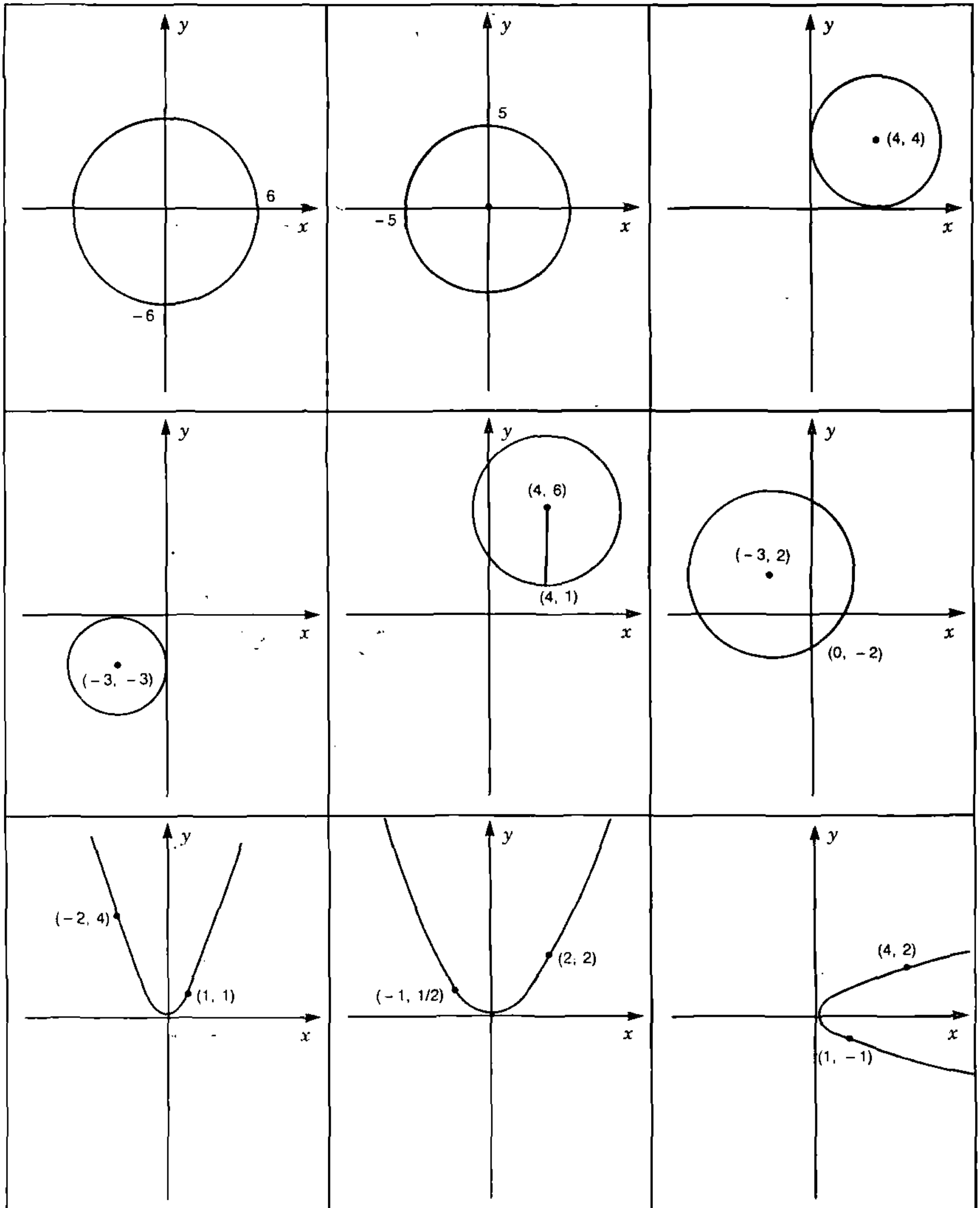
$\frac{x^2}{4} - \frac{y^2}{25} = 1$

$\frac{y^2}{4} - \frac{x^2}{9} = 1$

"Crazy Conic" Playing Cards




y
x
(1, −1)
(−2, −4)
y
x
(−√3,0)
(√3, 0)
(0, −3)
y
x
(0, 9)
(−3, 0)
y
x
(4, 0)
(2, −4)
y
x
4
6
y
x
2
6
y
x
2
4
y
x
16
10
y
x
6
4 1/2


y
x
6
−6
5
−5
(4, 4)
(−3, −3)
(4, 6)
(4, 1)
(−3, 2)
(0, −2)
(−2, 4)
(1, 1)
(−1, 1/2)
(2, 2)
(4, 2)
(1, −1)

54.

I Have . . . Who Has . . . ?

Topic: Coordinate geometry

Level: Grades 10–12

Number of Players: Whole class

Materials: Set of "I Have . . . Who Has . . . ?" cards (1 card for each student)

Procedure:

1. Shuffle the cards, and hand 1 to each student in the class until the deck is exhausted. Two cards can be given to some students. This will keep them in the game longer, so be selective.
2. Choose a student to begin. Suppose the student has the first card below. The student reads, "I have (3, −2)," pauses, and then reads the remainder of the card, "Who has the slope of $2x - y = 8$?"

I have	I have
(3, −2)	2
Who has	**Who has**
the slope of $2x - y = 8$?	the y−intercept of $2x - 3y = 9$?

3. The student with the second card answers, "I have 2," pauses, and then reads the question, "Who has the y−intercept of $2x - 3y = 9$?"
4. The activity continues in this manner until all students have read their cards. If some students are not included, it is because a wrong answer was given and went unchallenged. Let the students find where the incorrect answer occurred.

I have $y - 4 = x - 1$ **Who has** the coordinates of the point of intersection of $x = -3$ and $y = 4$?	**I have** $(-3, 4)$ **Who has** the equation of the line through (1,4) and slope -1?	**I have** $y - 4 = -1(x - 1)$ **Who has** the equation of the median to $\overline{AC}$ in $\triangle ABC$ for A(4,3); B(2,1) and C(6,1)?
I have $y - 2 = \frac{1}{3}(x - 5)$ **Who has** the intersection of the line $y = 5$ with the x − axis?	**I have** There is no y−intercept **Who has** x−intercept of the line $x = 9$?	**I have** (9,0) **Who has** the equation of a line with no slope through the point (4,9)?
I have $(3, -2)$ **Who has** the slope of $2x - y = 8$?	**I have** 2 **Who has** the y−intercept of $2x - 3y = 9$?	**I have** -3 **Who has** the midpoint of (3, 4) and (9, −8)?

I have

$\frac{3}{5}$

Who has

the equation of a line with a slope 2/3 and a y-intercept of 2?

I have

$2x - 3y + -6$

Who has

the equation of a line with a slope 2 and through (0, −5)?

I have

$2x - y = 5$

Who has

the slope of a line parallel to the line $x + 3y = 8$?

I have

$-\frac{1}{3}$

Who has

the inclination of a line with slope −1?

I have

135°

Who has

the slope of a line with an inclination of 30°?

I have

$\frac{1}{3}\sqrt{3}$

Who has

the equation of a line through (2, 5) with slope 1/3?

I have

$y - 5 = \frac{1}{3}(x - 2)$

Who has

the equation of a line through (2,3) and perpendicular to $2x + y = 7$?

I have

$y - 3 = \frac{1}{2}(x - 2)$

Who has

the equation of a line through (2,3) and parallel to $2x + y = 10$

I have

$y - 3 = -2(x - 2)$

Who has

the equation of the perpendicular bisector of $\overline{AB}$ if A(2,3) and B(0,5)?

I have $\frac{3}{2}$ **Who has** the equation of the line perpendicular to the x−axis through (−2, −4)?	**I have** $x = -2$ **Who has** the slope of a line perpendicular to $4x - y = 8$?	**I have** $-\frac{1}{4}$ **Who has** the x−intercept of $4x - y = 8$?
I have 2 **Who has** the equation of the perpendicular bisector of $\overline{AB}$ if A(3,4) and B(−1,2)?	**I have** $y - 3 = -2(x - 1)$ **Who has** the slope of a line with an inclination of 120°?	**I have** $-\sqrt{3}$ **Who has** the midpoint of $\overline{AB}$, if A(0,6) and B(4,8)?
I have (2, 7) **Who has** the slope of $3x + 2y = 6$?	**I have** $-\frac{3}{2}$ **Who has** the y−intercept of $3x + 2y = 6$?	**I have** 3 **Who has** the slope of a line perpendicular to $5x + 3y = 8$

I have $x = 4$ **Who has** the slope of the altitude to $\overline{BC}$ in △ ABC if A(1,6) B(−2,−8); and C(−7,−2)?	**I have** $\frac{5}{6}$ **Who has** the equation of a line through (7,2) and perpendicular to $x + 3y = 2$?	**I have** $y - 2 = 3(x - 7)$ **Who has** the equation of a line through (2,3) and parallel to the x−axis?
I have $y = 3$ **Who has** the coordinates of the vertex of the right angle in △ ABC, if A(−5,0); B(8,6); and C(3,−4)?	**I have** (3, −4) **Who has** the coordinates of the intersection of $x = 3$ with $x + 3y = 6$?	**I have** (3, 1) **Who has** the coordinates of the intersection of $2x + 3y = 8$ with the x-axis?
I have (4, 0) **Who has** the coordinates of the intersection of $x + 2y = 6$ and $x - 2y = 4$?	**I have** $(5, \frac{1}{2})$ **Who has** the midpoint of $\overline{AB}$ if A(7,2) and B(5,4)?	**I have** (6, 3) **Who has** the slope of $\overline{AC}$ if A(3,4) and C(1, 1)?

55.

Old Poly

Topic: Multiplication of polynomials or factoring of polynomials

Level: Grades 8–10

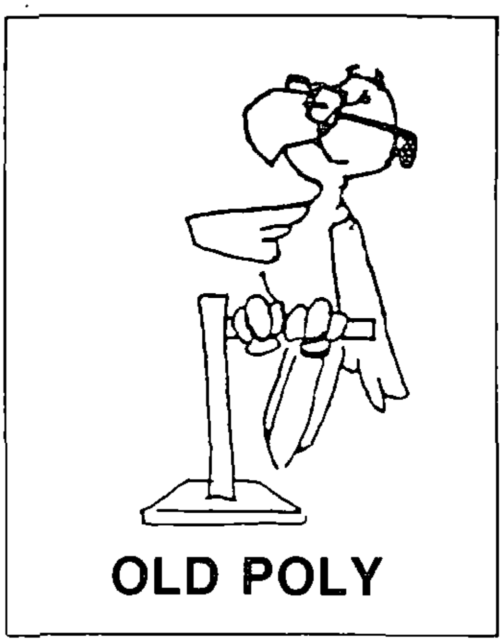
Number of Players: 3–4

Materials: Set of "Old Poly" playing cards

Procedure:

1. The dealer shuffles the cards and deals them all out. All players match the polynomial with its factors for those pairs in their hand. These pairs are placed face up on the table.
2. Players then pass 3 of the remaining cards to the player to their right. If new pairs are formed from this action, players add these to the spread in front of them.
3. To begin the draw, the player on the left of the dealer draws a card from the hand of the player to his or her left. If the drawn card completes a pair, the player plays the pair face up with the others. Otherwise, the player keeps the card, and the next player to the left draws from the player to his or her left.
4. Play continues until all the pairs are formed, leaving 1 player with the "Old Poly" card. This person is the loser. Players drop out of the game as their cards are depleted.

Variation: The player left with the "Old Poly" card is the winner!



$(a - 2)^2$

$(a - 2)^2$

$(p + 8)(p - 8)$

$(p + 8)(p - 8)$

$(a - 5)^2$

$(a - 5)^2$

$(p + 7)(p - 7)$

$(p + 7)(p - 7)$

$p^2 - 64$

$p^2 - 64$

$a^2 - 10a + 25$

$a^2 - 10a + 25$

$p^2 - 49$

$p^2 - 49$

$(c + 2)(c - 2)$

$(c + 2)(c - 2)$

$c^2 - 4$

$c^2 - 4$

$(5x + 4y)(5x - 4y)$

$(5x + 4y)(5x - 4y)$

$(a - 3)^2$

$(a - 3)^2$

$(5n - 1)^2$

$(5n - 1)^2$

OLD POLY


$25x^2 - 16y^2$

$25x^2 - 16y^2$

$a^2 - 6a + 9$

$a^2 - 6a + 9$

$25n^2 - 10n + 1$

$25n^2 - 10n + 1$

$25n^2 + 30n + 9$

$25n^2 + 30n + 9$

$(5n + 3)^2$

$(5n + 3)^2$

$(p + 5)(p - 5)$

$(p + 5)(p - 5)$

$4p^2 - 9$

$4p^2 - 9$

$(x + 2)^2$

$(x + 2)^2$

$b^2 - c^2$

$b^2 - c^2$

$(2p + 3)(2p - 3)$

$(2p + 3)(2p - 3)$

$x^2 + 4 \times 4 + 4$

$x^2 + 4 \times 4 + 4$

$(b + c)(b - c)$

$(b + c)(b - c)$

$(3x - y)^2$

$(3x - y)^2$

$(2a + 1)^2$

$(2a + 1)^2$

$9p^2 - 25$

$9p^2 - 25$

$9x^2 - 6xy + y^2$

$9x^2 - 6xy + y^2$

$4a^2 + 4a + 1$

$4a^2 + 4a + 1$

$(3p + 5)(3p - 5)$

$(3p + 5)(3p - 5)$

$16a^2 - 8a + 1$

$16a^2 - 8a + 1$

$(4a - 1)^2$

$(4a - 1)^2$

$p^2 - 25$

$p^2 - 25$

$4p^2 - 25$

$4p^2 - 25$

$(x + 6y)^2$

$(x + 6y)^2$

$x^2 + 12xy + 36y^2$

$x^2 + 12xy + 36y^2$

$x^2 - 1$

$x^2 - 1$

$(x - 1)^2$

$(x - 1)^2$

$b^2 - 16$

$b^2 - 16$

$(x + 1)(x - 1)$

$(x + 1)(x - 1)$

$x^2 - 2x + 1$

$x^2 - 2x + 1$

$(b + 4)(b - 4)$

$(b + 4)(b - 4)$

$(2p + 5)(2p - 5)$

$(2p + 5)(2p - 5)$

$a^2 - 4a + 4$

$a^2 - 4a + 4$

56.

Slope Speedway

Topic: Algebra

Number of Players: 2 players or teams of 4 or less

Objective: To introduce slope as the ratio of the change in y to the change in x.

Materials: Gameboard, 2 different-colored pencils, paper on which to keep a record of the moves

Procedure: Each player puts a dot on the x axis at Start (keeping in the bounds of the speedway). Players take turns. The move is determined by the ratio of the given slope. Each player begins with a slope of 1/1.

On each move, a player may increase or decrease the numerator (change in y) or the denominator (change in x) by 1. Thus, the first move may be chosen from 1/2, 2/1, 0/1, and 1/0. If 1/2 is chosen, the player moves up 1 and over 2 from the starting point. The player makes a choice and graphs the new coordinate point. A line segment is drawn to connect points in order.

Each player keeps a record of the choice of move (slope) on a sheet of paper. On subsequent turns, players alter the slope from the previous play by increasing or decreasing the numerator or denominator.

The first player to reach the finish line wins. A player may not move to an occupied point of the graph and is disqualified if the move results in a point on or outside the boundary of the track. If a player is disqualified, the other player wins.

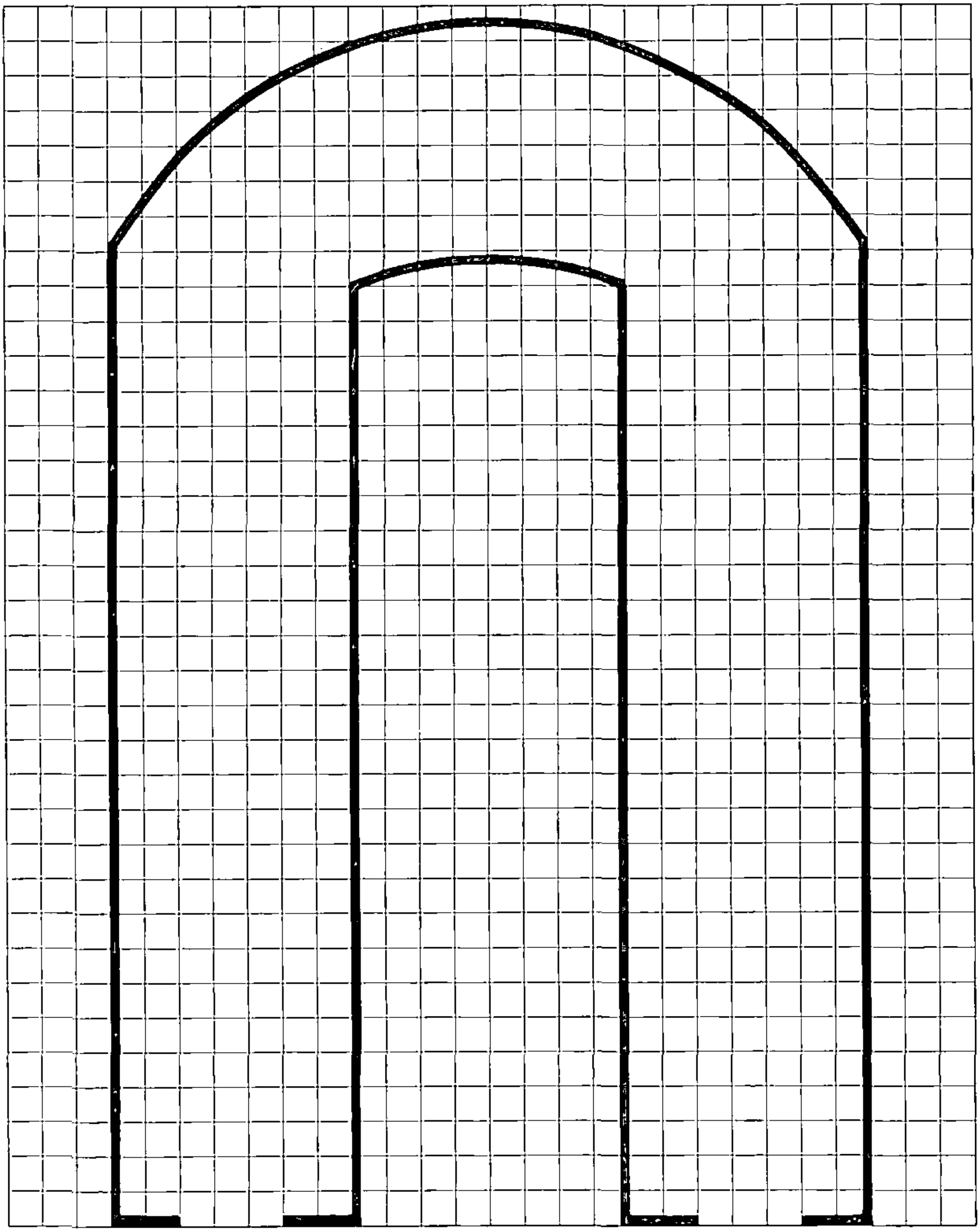
Sample Record Card:

Player 1	Player 2
2/1	1/2
2/2	1/1
1/2	0/1

NOTE: To proceed down the right side of the track, the y value must be a negative number.

Variation: To rule out undefined slope (vertical movement) because of division by zero in the fraction form, change the increase-decrease rule so that a decrease of 1 in the denominator (when the denominator is 1) becomes -1, not zero.

Slope Speedway



START FINISH