The Tuesday Problems

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In the Agenda for Action published by the NCTM, the number one recommendation for the 1980s suggested that problem-solving be the focus of the school mathematics program. St. John's-Ravenscourt School has had a formal problem-solving program since the fall of 1975. Here, in brief, is how our program is conducted.

At the Grade 8 and 9 level, we spend one day a week on problem-solving. Each week, we present the students with a set of eight problems that require various strategies to solve. Most of the problems are not directly related to the course work. The students have a week to solve them. When the week is up, we spend a period discussing them. The discussion period is a sharing of ideas, since many students find interesting and innovative ways of attacking some of the problems. It is important for them to realize that a given problem can be solved in a wide variety of ways.

The eight problems are carefully selected. One or two of them should be simple enough that every student will be able to solve them. The others should range from moderately easy to very difficult. Occasionally, there will be a problem that no student manages to solve. As long as students understand that perfection is not required, they do not object to this. In fact many of them tell us of the satisfaction that comes with finally cracking a problem they had been struggling with for days. To encourage the weaker students, we are generous with our praise for those who manage to solve even four out of the eight. It is also important, as the program advances, to recycle some of the ideas from the earlier problems. This allows the students to build on what they have learned.

We have whimsically named our problems after the days of the week. We started with *The Tuesday Problems* for the top stream of Grade 8s, and carried on the following year with *The Monday Problems* for the top stream of Grade 9s. Both problems series have been very successful and are still going strong. Our problem-solving at the senior secondary level has been less formal. The *Wednesday Problems* for Grade 12s were successful, but had to be cut back, since we could not afford to take one day a week from our regular curriculum. As a result, problem-solving in Grades 10 to 12 is done on a less regular schedule, usually two or three days at a time several times a year. We are also developing a problem series for our regular stream of Grade 8s.

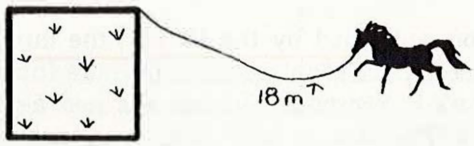
Several sample problem sets follow.

The Monday Problems

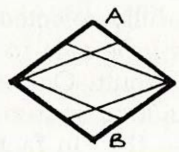
1. If $a*b = ab + 1$ and $a^\circ b = a + b + 1$, what is the value of $4*[(6^\circ 8)^\circ(3*5)]$?
2. Given the integers one to nine inclusive, how many ways can the sum 15 be obtained by taking them three at a time. Repetitions are not allowed.
3. Starting at point S on a circle, successive areas of $27\frac{1}{2}^\circ$ are marked off in one direction. If the first point obtained is called P_1, the second point P_2, the

thirds point P_3, and so on, what is the value of n if P_n is the first point to coincide with S?

4. Think of a number (this could be called x). Add 12. Multiply by four. Subtract 36. Divide by two. Write down two additional steps that will always get back to the original number.
5. A horse is tethered to a rope at one end of a square corral (outside the corral) 10 m to a side. The rope is 18 m long. What is the grazing area available to the horse?



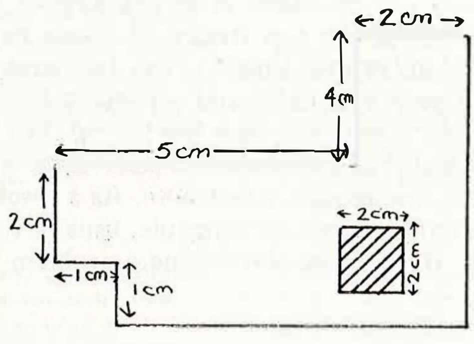
6. A contractor has a large number of pipes 2 m in diameter. He first makes a row of pipes side by side, and each is in contact with the next on a level surface. Then a second row is placed on top of this row so as to fit into the hollows between adjacent pipes. He continues this process until he has five rows. What is the total height of the pile of pipes? Express your answer in metres correct to one decimal place.
7. If only downward motion is allowed, find the number of paths from A to B in the following figure.



8. In the sequence 1, 3, 5, 2, 4, 6, 3, 5, 7, 4, 6, 8,...write the next few terms. Then find the sum of the first hundred terms.

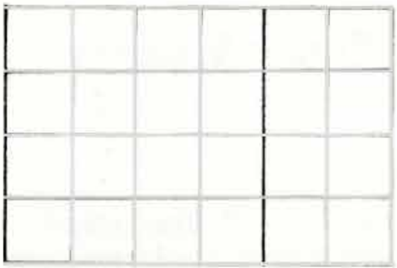
The Tuesday Problems

1. Find the area of the region shown below. The diagram is not drawn to scale. (All edges meet at right angles.)



2. Evaluate: $[\sqrt{.04} + \frac{1}{0.2} - \frac{6}{5}]^2$
3. a) Find the number half way between 1/3 and 4/5.
   b) Find the two numbers that divide the difference between ½ and 5/6 into three equal parts.
4. A 6 x 4 rectangle is divided into a number of equal-sized small squares as shown.

Find the total number of squares of all sizes in the diagram.



5. a) What is the value of: 100 - 99 + 98 - 97 + 96 - 95 + ... + 4 - 3 + 2 - 1
   b) What is the value of: 100 + 99 + 98 + 97 + 96 + 95 + ... + 4 + 3 + 2 + 1
6. How long will it take two water pipes to fill a 260 L tank if one pipe delivers 5 L in one minute and the other delivers 1 L in five minutes?
7. What is the least positive integer that 180 should be multiplied by if the product is to be a perfect square? What is the smallest positive integer we should multiply it by if we wish the answer to be a perfect cube?
8. If a student were to calculate the product of all the natural numbers from one to 25 inclusive (calculate 1 x 2 x 3 x 4 ... x 24 x 25), how many zeros would there be at the end of this result?

The Wednesday Problems

1. Factor: $x^2 + 5xy + x - 3y - 24y^2$.
2. If a and b are positive numbers such that: $\log_c \frac{1}{2}(a + b) = \frac{1}{2} \log_c a + \frac{1}{2} \log_c b$ show that a = b.
4. A sequence starts with three and ends with 71. There are five numbers in between. The law of the sequence is such that each term (after the first two) is the sum of the two preceding terms. Find the sequence.

   3, , , , , , 71

5. Graph the equation: $| y | + | x + 3 | + | x - 3 | = 10$
6. AB is the hypotenuse of a right triangle ABC. Medians AD = 7 and BE = 4. Find the length of AB.
   a) The base of a triangle is of length b and the altitude is of length h. A rectangle of height x is inscribed in the triangle with the base of the rectangle in the base of the triangle. Find the area of the rectangle in terms of b, h, and x.
   b) Prove that the largest rectangle that can be thus inscribed has area equal to one-half the area of the triangle.
7. A circle with centre C is given and eight distinct points are marked on the circumference so that no two points are on opposite ends of the same diameter. Prove that, no matter where the points are marked, it is possible to label two of them A and B so that three of the points lie on the smaller arc from A to B.
8. An escalator (moving staircase) of n uniform steps visible at all times descends at a constant speed. Two boys, A and Z, walk down the escalator steadily as it moves. A negotiating twice as many escalator steps per minute as Z. A reaches the bottom after taking 27 steps while Z reaches the bottom after taking 18 steps. Find the value of n.