Sines, Cosines and Ferris Wheels

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In *Using Advanced Algebra* (Travers *et al*, 1977), the textbook used in the Newfoundland Grade 11 matriculation program, a ferris wheel is presented in the introduction to the chapter on trigonometry. Such an analogy triggered a possible inductive approach to a discussion of trigonometric ratios, especially sine and cosine. This method has proven successful for me in three successive years of teaching, and its possibilities and objectives can vary. It could be oriented as a classroom demonstration or as group projects or as individual activities. Other circular analogs, besides the ferris wheel, could be considered. The length and depth of the project depend on your students' abilities and your own imagination. The suggestions below can be easily modified to meet your own teaching style.

The objectives of such a lesson may be to provide students with an opportunity to "discover" how the sine ratio (or cosine) can be developed; to investigate the need for constancy of these ratios; to experience "real" measuring events that ultimately lead to a rational conclusion.

The materials needed for the project:

1. A picture of a ferris wheel (if possible) or whatever circular rotating object you choose.
2. Bristol board (at least one sheet) to be used for modelling the ferris wheel.
3. Scissors, markers, and a thumbtack (or fastener).
4. Straightedge, protractor, large compass (board compass).
5. String (at least a metre); tape (masking is fine).

The only prerequisite for this activity is that students be familiar with the measuring tools (a metric ruler and a protractor) to be used.

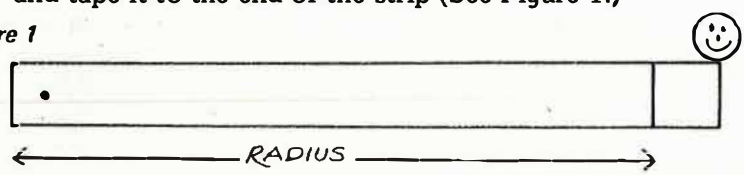
The preparation may be done by a group of students before the class, or during the class if desired. The purpose of the picture of the ferris wheel is to permit familiarization with it; not all students in Newfoundland have seen one. It should be posted in the classroom, if possible.

To build a model of a ferris wheel, follow these instructions:

1. Using the chalkboard compass (or a thumbtack, string and marker), draw a large circle on the sheet of Bristol board. Its size is limited only by the dimensions of the material.
2. Using a marker, indicate the centre point, and draw a diameter.
3. Cut out the circle.
4. From the remaining Bristol board, cut a strip one cm wide (at most) and let the strip's length be the same as the radius of the circle. Extend it "a little" to allow for later fastening.
5. To represent a rider on the ferris wheel, cut a small square with a circle atop,

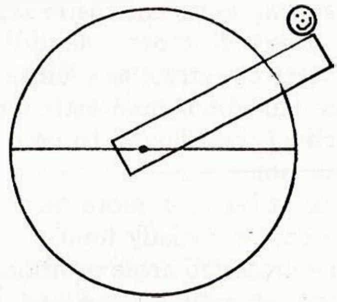
and tape it to the end of the strip (See Figure 1.)

Figure 1



6. Using the thumbtack or fastener, attach the "free" end of the strip to the centre point of the circle, so that the rider touches the edge of your circle. Tape the thumbtack in place to permit the strip to rotate (see Figure 2). The strip should rotate freely around the circle. If desired, it could be temporarily taped in a position.

Figure 2



A description of the inductive activity is presented below. The comments made are suggestions only.

1. Place the model of the ferris wheel in a convenient spot. The edge on the blackboard may serve useful. Be sure the diameter is in the horizontal position.
2. You might wish to demonstrate the rotation ability of the strip and discuss the analogy a little. For example, students may not realize that on the real ferris wheel, the seat of the rider adjusts, so that he/she is never upside down as our rider may be in certain positions.
3. Prepare for measurements. I have used the metre stick and a piece of string. The string should be cut the same length as the radius, since no measurement will exceed this. A protractor (for demonstration purposes, the larger board protractor) should be used. Other measuring tools such as a ruler or one's hands may also help cement the relationship to be developed.
4. Once the measuring tools are available, the induction should begin. A brief explanation may be necessary. Students should be told that they are about to measure the vertical distance of the rider in different positions on the ferris wheel. The vertical height will be measured from the fixed horizontal diameter, and the position will be recorded in angle measures. We will be looking for patterns.
5. Choose a beginning angle—this could be suggested by students or the teacher. Say 30° was chosen. Rotate the rider who should initially lie on the fixed diameter, to 30° above that line. Check the rotation measure with the protractor. Indicate, or have a student indicate, what would be considered the vertical distance. The idea of a perpendicular distance should come up and may need to be discussed.
6. Before making measurements, attach the rider in this position and briefly describe ways of recording data. A table of measurements is helpful in organizing the data. A partial table is given in Figure 3.

Angle	Verticle measure in cm	Verticle measure in string units
30°	7.5	0.50
50°	11.5	0.75
etc.	etc.	etc.

Figure 3

7. Now, measurements can begin. Ask a student to measure the vertical distance of the rider above the diameter using the metre stick. The centimetre will be the unit of measurement. Next, a student should measure the same distance with the string. Here, one sees the string as a unit, and the measures will be a fraction of the string. The student should estimate the fraction of the string needed. Remember the string is considered to be one unit or one radius. Now record your measures in the table.
8. Continue this procedure for at least six more measures. Alternate measures so all quadrants will be covered, or initially limit yourself to the first quadrant. The 60°, 45°, 90°, are some suggested angle positions.
9. Once the data are collected, an examination and a discussion are necessary. The teacher should permit students to guess and speculate as much as possible as to any possible relationships. If none is indicated, suggest measuring the radius of the circle with each instrument; record this at the top of each column. Now, suggest that this measure may help indicate a relation. It is hoped that the discussion will lead to the idea that the ratio of the vertical distance to the radius is the same, whichever unit of measurement is used. To develop the notion of cosine, use the horizontal distance from vertical diameter.
10. Check suggestions or guesses with further measures. Lead into the naming of such a measure as the sine function.

Activities such as presented above require at least one full class period. The use of this amount of time in the short term saves time in the long term, since students usually develop a better understanding of the concepts being taught. Have a good ride on your ferris wheel.

Reference

Travers, K.J. *et al. Using Advanced Algebra*, Doubleday, Canada Limited, Toronto 1977.