enter a_{n-2}, equals, etc., then 5n+2 by steps are required. However, on the HP calculators, only 3n+5 steps are required. If we had used the brackets, then 6n+1 key steps are required, so we might as well have evaluated f(x) in the more straightforward way described earlier.

In any case, you should get to know your particular calculator, because you may be able to discover shorter methods of evaluating f(x). For example, using a TI-35, one can evaluate fourth- and lower-degree polynomials by writing f(x) in the form: $x \cdot (x \cdot (x \cdot (x \cdot a_4 + a_3) + a_2) + a_1) + a_0$, then the argument x need not be stored, since it remains in the register as "times" and "left bracket" are pressed. No higher degrees can be evaluated in this way, since there can be, at most, four pending operations on the TI-35.

On some calculators, e.g., Casio College fx-100, "times" can be omitted before "left bracket."

Using synthetic division to evaluate f(x) will not result in fewer key steps, since this method is exactly the same as the nested form above, requiring 5n+2 key steps.

Students should be given the opportunity to approximate irrational roots of polynomials. Finding rational roots requires learning some very nice algebra, but it is very limited in the polynomials selected. It would also help us in Calculus I at university if students could solve the equation $f^1(x) = 0$, where $f^1(x)$ is a polynomial of degree three or more, and where the roots need not be rational.

References

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Probability and Statistics Corner

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This first article for the year of 1982 coincides with another first, the first issue of the *Canadian Mathematics Teacher*. I hope that PS Corner will become a regular feature of the journal, as it has in *Vector*, the journal of the B.C. Association of Mathematics Teachers.

At the recent Leadership Conference on Statistics in the Classroom, organized by the ASA/NCTM Joint Committee on the Curriculum in Statistics and Probability, I had the pleasure of meeting teachers from British Columbia, Alberta, Ontario, and Nova Scotia who were involved at some level of teaching statistics. All of those provinces are moving strongly toward a greater emphasis on statistics and probability in all levels of the curriculum. The majority of those present at the leadership conference were working at, or had a major interest in, Grades K-8. Now that it is generally accepted that future revisions of the mathematics curriculum must "recognize statistics and probability as important and indentifiable topics of the curriculum" (from the recommendations following the recent mathematics assessment in B.C.), we must determine the components of those topics and their place in the curriculum.

The following ideas are offered as the starting point of a discussion that will, I hope, produce a curriculum in statistics and probability. That curriculum should capture the investigative spirit of the subject and avoid the pitfalls that have produced the present situation of topics being on a syllabus, but avoided by most teachers.

The essential elements of an elementary school program:
Collecting numerical information.
Exploring the visual representations of such data, pictographs, stem and leaf graphs, indeed any kind of visual presentation that assists with asking and answering questions.
Developing skills in asking questions of such data; such skills will also develop the use of such concepts as *greater than, greatest, how many are less than...*, *range* (greatest–least), and variability.
Basic concepts of probability, laying the framework for the later development of the idea of a simulation. Use of a variety of games to develop the idea of a fair game as opposed to an unfair game.
Two co-ordinate graphs are introduced at this level and can be used to develop ideas of variability through an examination of such graphs as height vs. weight or span vs. height, etc.

Two emphases could predominate a program for Grades 7-9:
The concept and use of simulation. This is a powerful problem solving tool and valuable for the reinforcement of the ideas of probability and variability.
Exploring data, coupled with a project approach. Calculation of means, etc., is a pointless exercise if the process stops at the calculation. Tying these skills to a topic of interest to the student (sports would be of interest to a large number of students) gives some point to the exploration of data. Interest in the subject is very important if this area is to be learned successfully. It should, perhaps, be obvious to a number of you that these two topics lend themselves to integration with computer literacy. Computers should play an important role at this level, if not earlier levels. The computer will make possible tasks that would otherwise be tedious or time consuming.

Other areas that are appropriate for this level are precision of measurement and errors; simple sampling experiments, sample bias and biased sampling, reading tables; and reading and interpreting newspaper articles.

In a recent editorial by Peter Holmes in *Teaching Statistics*, he makes a relevant point: Where in the curriculum should we develop the skills necessary for students to be able to answer questions such as "Should seat belts be made compulsory?"
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In the United Kingdom at the moment there is a growing debate about numeracy. How numerate are our pupils? The first reference to the word "numeracy" that I can trace is in the Crowther report (1959) where it was used in the context of the ability to reason quantitatively and to have some understanding of the scientific method. In this sense to be numerate is to be able to use numbers in drawing inferences, the very essence of statistics. More recent definitions have drawn the difference between algorithmic skills and informed numeracy. Having algorithmic skills means having the ability to do sums; informed numeracy means the ability to know what sums to do, when to do them and how to use the answer. Another recent definition of numeracy talks of "the ability to use number in practice." Clearly all these definitions relate closely to the statistical education we should be incorporating into the main secondary school curriculum. Perhaps we need to rethink carefully our basic statistical material for schools, asking ourselves, "Does this help pupils develop the ability to use number in practice?" As an example consider the following question from a national examination in English to 16-year-old pupils of average ability.

Read this extract from a booklet published by the West Yorkshire Metropolitan Police:

USE OF SAFETY BELTS
drivers and front seat passengers in cars and light vans

		CASUALITIES		
SAFETY BELT	KILLED	SERIOUS	SLIGHT	TOTAL
Fitted and worn	4	72	447	523
Fitted but not worn	62	612	2543	3217
Not fitted or not known	2	43	132	177
Total casualities	68	727	3122	3917

...and those figures tell their own story, don't they?

Discuss whether the wearing of seat belts should be compulsory by law.

How well has our teaching enabled pupils to answer questions like this? Do they know what to look for in the table, what they might find, how to allow for variability in interpreting their answer, and so on? We owe it to our pupils to prepare them for this type of use of number in practice.

The program for Grade 10-12:

In my experience, the project or experimental approach is the most effective way to convey the idea that statistics is experimental and investigative. The subject matter at this level therefore develops investigative skills. It includes such things as survey sampling, the idea of a reference distribution applied to YES/NO samples, measurement samples, and samples of the chisquared statistic. The chisquared material should not be a formula that is interpreted using a table, but a means of developing the concepts of expectation and a reference distribution. The elements of interpretation of scatter plots is also appropriate at this level.

So much for a start as to what should be in the curriculum. I have not addressed the issue of content for the range of courses at the senior levels, consumer math, statistics courses, computer science, etc., nor have I addressed the issue of what should be omitted. Some people might note certain omissions, such as standard deviation and the normal distribution. Such things will, I hope, be considered in the ensuing discussion.

Teaching Problem-Solving

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The history of mathematics education shows that problem-solving has always been an issue of concern. Most recently, the National Advisory Committee on Mathematics Education (NACOME) and the National Council of Supervisors of Mathematics (NCSM)[1] have indicated that problem-solving is one of the most important basic skills that every student should master in order to survive in our society. The National Council of Teachers of Mathematics (NCTM) has also taken a position supporting the infusion of problem-solving into the school mathematics curriculum for the 1980s.[2] However, there is a great deal of misunderstanding over what problem-solving is and even what constitutes a problem. Jeremy Kilpatrick[3], in his article entitled "Stop the Bandwagon, I Want Off," decries the "use of 'problem-solving' as an empty vessel that we can fill with our own meanings." In many cases, mathematics teachers teach solving problems with the mistaken idea that they are teaching problem-solving.

This article has basically two purposes: to illuminate and reflect on problem-solving and to illustrate, by way of an example, how to teach problem-solving skills in a classroom.

What is a problem?

One common concept of a problem is that of a question proposed for an answer or solution. The teacher has this concept in mind when he/she says to a mathematics class, "Your assignment for tomorrow is to work problems one to ten on page 164." The question that may be either *explicit* or *implicit* in each problem is, "What is the answer?"

A second concept of a problem considers the existence of a question to be necessary, but, unlike the first concept, existence of the question is not regarded as sufficient. The additional conditions pertain to the individual who is considering the questions. What may be a problem for one individual may not be a problem for another. A problem for a particular individual today may not be a problem tomorrow.

The necessary conditions for the existence of a problem for a particular individual are:[4]