Individualizing Instruction Through Multilevel Performance Problem-Solving Activities

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One of the greatest challenges facing teachers of mathematics in elementary and middle schools lies in providing for the needs of learners with differing abilities and interests. That individual differences among students exist at all grade levels and that the range of these differences increases from grade level to grade level is common knowledge among teachers and has been well documented by research. Jarvis (1964), for example, found that Grade 6 students may vary by as much as seven years in arithmetic achievement, and that even among students with IQs of 115 or higher, the range of achievement is about five years.

Several procedures for accommodating individual differences in mathematics have been explored over the years. One approach that has received considerable attention is self-paced instruction. Self-paced or individualized programs are designed to enable each student to progress at his/her own rate of speed through a sequence of learning units. Research comparing the effectiveness of such programs with conventional teacher-directed settings, however, indicates that in general, and particularly in Grades 5 through 8, the self-paced approach has been ineffective in mathematics (Schoen, 1976). The lack of interaction among students and the reduced contact of students with their teacher are two of the major drawbacks of such programs.

Another organizational scheme at the school level for meeting individual differences is homogeneous grouping of students. Although differences among students will still exist, that approach can reduce the range of ability of achievement within a class, permitting the teacher to adjust instruction to suit the needs of more able, average, or less able students. Grouping on the basis of ability has been found in some studies to be effective, particularly for students at upper ability levels. Where grouping is based on achievement, research findings are more variable. It appears that the teacher is the most important factor in determining the success of any system of grouping (Suydam and Weaver, 1970).

Grouping may also be used within a self-contained class. For example, three instructional groups, studying different content and working at different levels, could be created. An alternative procedure involves flexible grouping with all students studying the same topics. Under this plan each new unit is introduced to the whole class. Following a period of instruction relating to the basic objectives of the unit, a diagnostic test is administered. Three groups—reteach, practice, and enrichment—are then formed (Underhill, 1972). The enrichment group, having mastered the basic material, studies related topics or investigates the content at a deeper level. Students from the three groups come together again for the unit culmination and the beginning of the next topic.

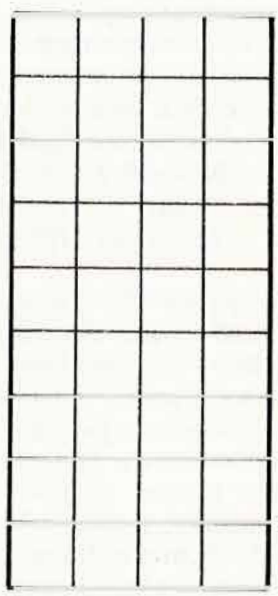
A great many teachers of mathematics in Grades 4 to 8 work with heterogeneous groups of students and do not employ self-pacing or grouping for instruction. The purpose of this article is to describe and illustrate a strategy for accommodating individual differences in mathematics within such a classroom setting. The suggested strategy involves the use of multilevel performance problem-solving activities.

Most, if not all, concepts and problems in mathematics can be presented and investigated at a variety of levels of sophistication. Thus, in teacher classes composed of students of mixed ability, it is not only desirable, but often possible, to provide learning activities that develop or reinforce the important concept or skill of the day's lesson and that permit each student to work at his/her level of understanding. To be appropriate for slow learners, the problem or game setting must allow each student to begin working immediately at some level. For example, the task might initially involve the use of concrete materials or require only basic counting techniques. To accommodate the mathematically talented, the problem could be open to more than one interpretation or have several methods of solution, and solving the problem would suggest to the student new problems to be explored. Capable students should also be encouraged to discover why a result holds and to generalize beyond specific cases.

I'll describe four learning activities that possess these characteristics. The first is a mathematics laboratory activity; the second, a setting for computational practice; the third, a game; and the fourth, a simulated "real-life" problem-solving project. For each activity, a procedure for presenting the task to the class is suggested, possible learning outcomes are described, and ways in which the activity might accommodate the needs of less able and more able students are discussed.

Activity 1. Area and Perimeter.
Materials. Thirty-six square tiles for each pair of students.

Teacher presentation and instructions
Using an overhead projector, the teacher arranges the 36 tiles to form a rectangular region as indicated. Terminology is developed/reviewed using this example. The *base* of the rectangle is (by counting) four units long and the *height* is nine units. The *perimeter* or distance around the outside of the rectangle is (counting) 26 units. The area, which is the number of unit squares covering the region, is (again counting) 36 square units.



The students are then instructed to make other rectangular regions, using all 36 tiles each time, and to find the perimeter and area of each, recording findings in a table as illustrated.

base	height	perimeter	area
4	9	26	36

Learning outcomes and individual differences
All students should be able to use the tiles to form another rectangular region and find its perimeter and area. Low-ability students usually require extensive work at

the concrete level and will likely continue to use the materials to form the various rectangular regions throughout the class period. Using basic counting techniques to find the perimeter and area will reinforce the meaning of these concepts and the difference between them. This experience will normally lead students to develop more efficient ways of finding the perimeter and area of a rectangle. They are often surprised, having "discovered" that area is base times height and perimeter can be found by doubling the sum of the base and the height (or taking double the base plus double the height), that these are the formulas they were previously taught. That the area is the same in all cases and the reason why this should be so are, for some students, non-trivial learning outcomes of this activity. Finding that the perimeter of the various rectangles is not constant surprises most students. It is usually noted that long, narrow rectangles have large perimeters and the square the smallest perimeter.

More able learners require less work at the concrete level and move quickly into abstract thinking. In this activity, capable students will likely put aside the tiles after using them to make one or two rectangles, and complete the table using symbolic procedures. The possible rectangles are related to the factors of 36, a formula is used to compute perimeter, and the area is seen immediately to be constant. These students may then be encouraged to draw graphs of the data; for example, they could plot base against height and perimeter as a function of base length. They could also go on to consider the perimeters of families of rectangles with areas of, say, 40, 26, and 31 square units, in order to generalize about the shape with the minimum perimeter for a fixed area.

An investigation of the areas of simple closed curves with a constant perimeter would be a natural follow-up activity. For example, if 20 m of fencing were to be used to enclose a garden plot, what shape would have the greatest area?

Activity 2. Practice and Patterns in Addition.

Teacher presentation and instructions
On the chalkboard, the teacher draws a 2 x 2 grid and writes a number in each of the four cells (Figure 1). With student participation, the numbers are added two at a time; first across, then down, and the sums written as indicated (Figure 2). Two loops are drawn on the upper corners of the grid, and in them are written the sums of the pairs of numbers along the diagonals (Figure 3). Next the two horizontal sums are added, and the answer is written underneath (Figure 4). The vertical and diagonal sums are also added, and it is noted (with appropriate flair) that the three answers are the same.

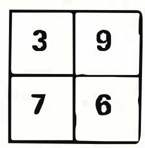


Figure 1

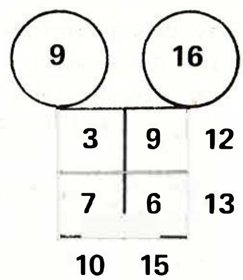


Figure 3

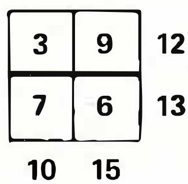


Figure 2

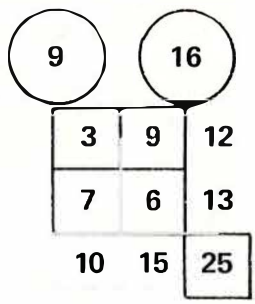


Figure 4

The question that should arise from this presentation is: "Does it always work?" Each student is asked to choose his/her own set of four numbers and try it again. The question "Why does it work?" should then be raised (by the class). After discussion, the setting is used to provide practice in addition. The numbers can be multi-digit whole numbers, decimals, fractions, or mixed numbers. Each student creates and checks his/her own set of exercises by selecting four numbers and following the procedure.

Learning outcomes and individual differences

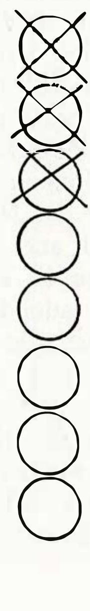
Working through this procedure requires the student to do nine addition questions, and each one must be done correctly in order for it to work. This task, however, appears less forbidding to the slow learner than a worksheet or textbook assignment of nine questions. Moreover the feedback is immediate and comes from the work itself rather than from an external authority. Pupils of lesser ability might at first choose easier numbers to work with, but having completed the task once, might be willing to try it again (nine more questions) with larger or more difficult numbers.

High-ability students can work with numbers requiring more complex computation and will perhaps do more sets of exercises in a given period of time than other class members. This setting, though, can be extended to provide worth-while mathematical activity beyond practice in addition, through consideration of "what if?" questions. More capable students can be encouraged to formulate and investigate such problems as "What if the operation is multiplication, or subtraction?" It still works for multiplication (and therefore provides a setting for practice in this topic). For subtraction, questions such as "Why doesn't it work?" "What does happen?" and "Under what conditions does a particular result (for example, two of the three differences are the same) follow?" can be considered. Thus the thrust of learning activity is shifted from computation to discovery, hypothesis-building, and verification.

Activity 3. Nim (a strategy game).
Materials. Eight counters for each pair of students.

Teacher presentation and instructions

The game with the eight counters is played by two people as follows. Taking turns, each player removes one, two, or three of the counters. The player who is forced to take the last of the eight counters loses.



Learning outcomes and individual differences

After varying amounts of experience in this game situation, students will discover that the player who has the first turn can always win by removing three counters on the first play. The logical strategy of considering all (in this case, three) possible moves by the second player and the corresponding subsequent moves by the first player can then by discussed as a "proof" for the winning strategy.

As previously stated, the point that the first player can *always* win and that chance is not a factor is understood by different students at different times during the activity, and it is clear to an observer when an individual actually gains an understanding of the situation. The teacher should not attempt to explain the logic to someone who has not discovered it, but should simply allow the student to continue to play the game until the idea is understood.

For those who discover and understand the winning strategy, the game is, of course, not a game. However, if any one of the variables is changed, then the process of determining a winning strategy must be repeated. Varying the game conditions as follows provides natural extensions of the learning activity.

1. The player who takes the last counter *wins.*
2. The number of counters is varied. For example, the game is played with nine, twenty-one, or fifty counters.
3. More than three counters (for example, up to four or five or nine) may be removed at each play.

Each combination of the above variables leads to a different winning strategy, but the logical precedure for determining the strategy is the same. Ask the class to establish conditions for one of these games, determine how to win it, and then demonstrate to the teacher that he/she can win every time. For example, a student might say to the teacher, "Start with 15 counters. Remove one to four counters each play. The person who takes the last counter wins. And you have to start."

Having determined the winning strategy under a variety of conditions, mathematically talented students could be challenged to begin to generalize the method. For example, if one to nine counters may be removed, and the person who takes the last counter loses, what should be the first move if you start with 60, 72, 87,... counters? Suppose three, four, five...counters may be removed on a play?

Activity 4. Shopping Spree.
Materials. Department-store catalogues, calculators.

Teacher presentation and instructions
The catalogues are distributed to the class—one to every student or pair of students. The students are told they may "spend" up to $400 to purchase clothing shown in the catalogue, assuming there is a 20% discount on prices listed and that a 6% sales tax must be paid. Since an implicit aspect of the task is to spend as close to $400 as possible (but not more), the procedure is repeated several times with different numbers by each student. In working on the task, two different problems emerge: (1) What is the maximum total list price? and (2) Can I find a desirable combination of items costing a sum close to this amount?

While this activity provides opportunities for practice in estimation, mental arithmetic, and computation, hand-held calculators should be permitted and their use encouraged. The problem-solving strategy of using successive approximations is encountered in exploring the first problem. Investigating the second problem provides students practice in formulating their own questions and identifying relevant data needed to answer them.

While doing this activity, students often generate alternative methods for calculating the final cost of the clothing. Consideration of these procedures can lead to some interesting mathematical questions. I'll list some of these:

1. Could you compute the final price (sale price plus sales tax) of each item separately and then add these figures? (Yes)
2. Could you simply subtract 14% (20%-5%) from the list price to get the final cost? (No)
3. Do you get the same final answer if you first add the 6% sales tax and then subtract the 20% discount as you get doing it the other way? (Surprisingly, yes.)
4. Could you compute the sale price in one step instead of two? (Yes, multiply by 0.8.)
5. Given the list price, could you compute the final cost in a single step? (Yes, multiply by 0.848.)

The equation, final cost = list price x 0.8 x 1.06, provides the answers to questions 2, 3 and 5 and also gives a way of finding the maximum list price directly:

$$\text{maximum list price} = \$400 \div 0.848 = \$471.70$$

To provide practice in solving verbal problems, have each student make up three story problems based on information in the catalogue. Encourage each student to create interesting and challenging questions, but to be able to answer them himself/herself from the information provided. After the students have prepared the problems and worked out an answer key, problem sets could be exchanged (between students of comparable ability). Solutions would be returned to the authors of the problems for marking.

Summary

The purpose of this article was to provide examples of multilevel performance problem-solving activities that can be used to accommodate individual differences among students in a heterogeneous whole-class instructional setting. Such experiences can enable slower learners to enjoy immediate success as they develop and practise basic concepts and skills. At the same time, more capable students may explore the topics in greater depth and engage in study involving higher-level mathematical processes. Participation in these class activities under the guidance of a teacher also provides valuable training that prepares the student for independent work on open-ended problems.

Many exercises and activities in school mathematics can be adapted by the teacher to allow for multilevel work. Two sources of appropriate materials are the "Ideas" section of the *Arithmetic Teacher* (Hirsch and Meyer, 1981) and the "Activities" section of the *Mathematics Teacher* (Hirsch, 1980). The use of multilevel performance problem-solving activities as a means of differentiating instruction in a whole-class setting can contribute in a positive way to the development of an instructional program that is *rich* for all students and *enriched* for more capable learners.

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Teach Nothing About Geometry

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Contrary to a likely interpretation of the title, I am not advocating the deletion of geometry from the mathematics curriculum. In fact, I am quite concerned about the near future of geometry in the curriculum and would not wish to see its position eroded any more than it is. I am concerned because the coming emphasis on and enthusiasm for computer literacy and microcomputer applications could easily push geometry further into the background, simply because geometry doesn't lend itself easily to micro-computer uses.

To return to the title, I am advocating the teaching of nothing about geometry in the sense of "no-thing." "No-thing" implies that we are not talking about a "thing." It is generally held that geometry instruction ought to include practice in space visualization, skills for organizing knowledge about space, attitudes favorable to local space exploration, and so on. But these are no-things that are about things. They are procedural skills, attitudes, or the seeing of relationships. The notion that no-things can be about things is crucial here, since the distinction between things and no-things is frequently the essence of arguments about the value of using geometric activities in the classroom. As an example, the "seeing of geometric relationships" might be acknowledged as an important mathematical goal, but none the less be slighted because it lacks a certain concreteness; for example, it is difficult to define as a teaching objective and is certainly difficult to test. None the less, a growing body of research indicates the existence of certain generalized skills and abilities that are important in problem-solving and applications. We ought to recognize these no-things of geometric activities and acknowledge their importance by insisting on their inclusion in the mathematics curriculum.

To further illustrate some of the points that I have been trying to make, I will describe and use a family of geometric activities. (Incidentally, these activities can easily be put into a game format if desired.) The activities will be defined, and references will be made to the no-things of geometry that they illustrate.

The game of "Turn a Pattern" (TAP)

(This is adapted from Marion Walter's *Boxes, Squares and Other Things.*) I will begin with a discussion of the rules for the two-dimensional version of the game: