Applications of Mathematics—Some Aspects

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Secondary mathematics, besides being a thing of many beauties and essential background for many of the students who "go on for higher studies," has direct applications to trades and occupations, personal finance, consumerism, homemaking, sports, and hobbies. Potential applications of secondary mathematics range from direct use of a single mathematical idea or technique to a fairly extensive theory-building of mathematics. However, applications of secondary mathematics are hard to teach, since they rarely occur in secondary mathematics. Teachers are uncomfortable with classroom applications of mathematics. Moreover, applications in mathematics texts are often represented by artificial "story problems," which normally appear at the end of every chapter in a mathematics text and are basically designed for reinforcement or enrichment of concepts already learned. Their use hardly convinces the student that mathematics has its applications in everyday life. Last, there is a great dearth of suitable instructional materials; so applications can become outdated faster than topics in "pure" mathematics.

Pamela Ames, of University of Chicago Laboratory Schools, has categorized mathematical applications into four broad levels:

Level Zero

These are a large collection (mostly mental) of very short statements consisting of references or allusions to uses of mathematics. They are used in class when we are dealing with a particular idea. Just refer to uses even if no applications are actually being considered or planned in a lesson.

Examples

(Lesson on vector sum) Airplane pilots use vector sums for every day trip.

(Lesson on congruence) Congruence is the basis of all mass production.

Most of the level zero statements are complete in themselves; once you have mentioned them, there is nothing more to say or do. The purpose is to keep the class in touch with the real world. Such statements do not arise automatically; you have to look for them and plan their appearance during instruction. Resolve to make one remark per unit or lesson. With effort and time, you can develop a large enough bank of these statements to make at least one such remark a day.

Level One

These are the so-called story or word problems. They are short, self-contained, artificial, real-world situations that usually pose a question that has a single solution or an easily obtainable definite number of solutions. Normally a "search model" is a linear or a quadratic equation which is readily available.

Textbooks and teachers do a fairly adequate job in this area though most of the problems are too artificial and remote.

Level Two

These are entire lessons built around a single real-world situation, and they may take from one to five class periods to complete. A single situation is investigated in depth using many different mathematical techniques. This level of application is the most important and the most neglected.

In this application stage, the plan is to work long enough within one situation to see mathematics as a resource to build understanding of the situation rather than mathematics as a tool to carve out answers to specific questions. An example of Level Two application may be:

—Building a garage or a swimming pool
—Making cost estimates or drawing floor plans.

Level Three

These are open-ended investigations. They are simply fertile ideas that I have not really dealt with yet. I have used few of these with a class, because I rarely take a whole class on an open-ended investigation. I save them for individuals who are ready to strike out alone or in small groups.

As a teacher, I place the situations that I would like to do something with, but when the ideas do get investigated, I get some Level Zero, One, and Two material from the results.

The purpose of this paper is to focus on *Level Two Applications,* since they are the most important and the most neglected in secondary mathematics. We can all accomplish this goal if we take a problem-solving strategy with a wide variety of applications and uses of mathematics. You have to look constantly for ideas. The two problems given below show how mathematics can be applied, problem-solving skills developed, and incidental learning in another area incorporated. As a by-product of this approach, students gain awareness of some of the environmental issues that confront our society today. Moreover, teaching secondary mathematics through applications will assure students that mathematics is all around us and there is no reason to fear mathematics.

PROBLEM #1

How to estimate the size of wildlife populations.

Some realizations of this problem

How many fish in a pond?
How many trees in a given forest area?
How many chips in an envelope?

Some ways to solve the problem

Count—This method is tedious, time consuming, and impractical.
Example: Catch all the live fish in a pond an count them. One way is to drain the water and count all the fish. The result? Dead fish. The end would not justify the means.

Reduce the problem to a situation that is solvable (closer to reality).

Defining parameters of the problem

How many chips in this envelope?
What is n? (n = no. of chips)

Do the following tasks:
Select 10 chips, mark them, and return them to the envelope.
Shake the envelope, select any 15 chips; record the number of marked ones, and return all of them to the envelope.
Shake the envelope and repeat (10 times in all).
Tabulate your observations.

A suggested solution
Defined variables:
n: Total number of chips in an envelope
y: Number of chips marked
q: Number of chips taken out each time
$\bar{y}$: Mean of the marked chips for x trials

Given y, q, and $\bar{y}$, we can calculate n using: $\frac{y}{n} = \frac{\bar{y}}{q}$

The table below the values for y, q and $\bar{y}$ for 10 trials.

TRIAL	NUMBER MARKED: y = 10	NUMBER SELECTED: q = 15
1	3	15
2	2	15
3	4	15
4	2	15
5	1	15
6	0	15
7	1	15
8	0	15
9	1	15
10	2	15
	TOTAL 16	
	$\bar{y}$ = 1.6	

Using $\frac{y}{n} = \frac{\bar{y}}{q}$, we have

$$\frac{10}{n} = \frac{1.6}{15}$$

$$\Rightarrow n = 93.75 \cong 94$$

The exact number of chips in the envelope was 100. The answer of 94 is a reasonable estimate. However, if the experiment is repeated a number of times, the answer will be closer to 100.

Can this problem help us to solve the original problem?
Surely, applying the same strategy on a pond problem or taking samples of the various regions of a forest can give us a reasonable estimate of the number of fish in a pond or the size of the wildlife populations in a forest.

PROBLEM #2

Mathematics of water conservation
Are we wasting part of the water supply? Let's look at the average daily water use per person. (See Table 1.)

TABLE 1
Average Daily Water Use Per Person

Purpose	Gallons	Litres
Flushing toilets	24.6	82
Washing, bathing	22.2	74
Kitchen use, drinking	6.6	22
Cleaning house, clothing	4.2	14
Washing car, watering garden	2.4	8
TOTAL	60.0	200

Adapted from Thomas R. Brehman. *Environmental Demonstrations, Experiments and Projects for the Secondary School,* West Nyack, New York: Parker Publishing Co., 1973, p. 141. (This table uses 1 litre = 0.3 gal.)

PART A
Some questions for class discussion:

1. What per cent of the total daily use of water is spent for each item?
   Which has the largest per cent?
   (Flushing toilets—41 %)

Which has the smallest percent? (Washing cars, gardening—4%)

2. Make a circle graph to represent the daily water use per person.

3. How could you conserve water?

4. How much water would be used daily in U.S.A., Canada, North American Continent, Europe, Asia, or world?

5. What per cent of the world's daily use of water is taken up by Canada, U.S.A., and North America?

Answers to some of these questions give information about one person or nation or country. Applied on a national or world level, the numbers are staggering and begin to give students a sense of the enormity of the problem.

PART B

Table 2 shows the estimated daily water use in the following categories from 1940-1980 in U.S.A.: Irrigation, Utilities, Domestic, Industrial.

TABLE 2
Estimated Water Use 1940-1980
(Billions of Litres)

Year	Irrigation	Utilities	Domestic	Industrial	Total
1940	269	126	12	110	516
1950	379	227	17	144	767
1960	511	457	23	232	1222
1970	451	560	16	211	1239
1980*	514	859	18	284	1675

*Projected

From *Statistical Abstract of the United States*. 96th Annual Edition. Washington, D.C.: Bureau of the Census, 1975, p. 179.

Some questions for discussion

1. What per cent of the water used in the U.S.A. in 1940-1980 was for various categories?

2. How do you account for the increase or decrease in water consumption? Why is the increase in utilities the greatest? What category has the least consumption?

3. If domestic users save water, will the total amount used be off sharply? Why?

Table 2 shows how much water is used for various purposes each day in the U.S.A. in billions of litres. Students can see from this table that the huge amounts of water tallied in considering daily water use per person are only a small part of the total water use!

PART C

Where does water supply come from? Table 3 gives the facts needed to compute each person's share of Earth's water. Are we using more water than our share?

TABLE 3
Facts Needed To Compute Each Person's Share of Earth's Water

Earth's water supply:
109,000,000,000 gal. = 4.12×10^{11}L.

World population:
3,616,000,000 people

Area of Earth covered by oceans:
139,356,000 mi.^{2} = 3.60×10^8 km^2

Average ocean depth:
12,451 ft. = 3.79 km

Per cent of Earth's water in ocean:
92%

The data in each of these tables is estimated and will vary with the source, which enables us to emphasize another important concept: Question the sources of information closely before using data to draw conclusions.

What concepts do students learn besides getting a feel for these vital issues of natural resource?

Multiplication and division of large numbers.
Ratio, per cent, and proportion.
Interpreting and using data.
Making and interpreting graphs.
Sampling techniques.
Problem-solving.

Additional thoughts for projects

Project #1
Locate a dripping faucet.

Measure in millilitres the amount of water wasted in 10 minutes.

Predict how much water is lost in an hour; in a day.

Find out what the water rates are in your community. With this information, calculate how much the drip costs in one year. Find out the repair costs. How long will it take for the washer to pay for itself by the water it saves?

Project #2
How much water do you waste while brushing your teeth or taking a shower?

Devise a measuring scheme.

Figure out how much water you waste in a week, a month, a year, for the whole family over the same time period.

Make a table and a graph of this information.

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