Strategies of Problem-Solving

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Students should learn and be able to choose procedures for solving problems. The procedures are easy to state and recognize. However, teachers have difficulty teaching problem-solving, because, unlike teaching computational skills or concepts, no specific content is involved. In problem-solving, an individually acquired set of processes is brought to bear on a situation that confronts students.

Four procedures (steps) are inherent in problem-solving. These procedures, their descriptions and associated strategies have been compiled and adapted from a variety of sources and authors (George Polya, J.F. LeBlanc, Ohio Department of Education, Math Resource Project, 1980, NCTM Yearbook) and are listed below:

STEPS IN PROBLEM-SOLVING

Understand the Problem

What is the problem? What are you trying to find? What is happening? What are you asked to do?

Strategies:

Paraphrase the problem or question (Students restate the problem in their own words to internalize the problem.)

Identify wanted, given, and needed information (Students focus on what has to be determined from the problem statement, and they list information so that they may discover a relationship between what is known and what is required.)

Make a drawing (Students depict the information of a problem, especially situations involving geometric ideas.)

Act it out (Students can picture how the problem actions occur and how they are related, thereby gaining a better understanding of the problem.)

Check for hidden assumptions (What precisely does the problem say or not say? Are students assuming something that may not be implied? Beware of mistaken inferences.)

Devise a Plan To Solve The Problem

What operations should students use? What do students need to do to solve the problem? How can students obtain more information or data to seek the solution?

Strategies

Solve a simpler (or similar) problem (Students momentarily set aside the original problem to work on a simpler or similar case. The relationship of the simpler problem should point to the solution for the original problem.)

Construct a table (Organizing data in tabular form makes it easier for students to establish patterns and to identify missing information.)

Look for a pattern or trend (Does a pattern continue or exist? In connection with the use of a table, graph, etc., patterns or trends may be more apparent.)

Solve part of the problem (Sometimes a series of actions, each dependent upon the preceding one, is required to reach a solution. Certain initial actions may either produce a solution or uncover additional information to simplify solving the problem.)

Make a graph or numberline (Students may organize information in such a way that it makes the relationship between given information and desired solution more apparent.)

Make a diagram or model (When using the model strategy students select objects or actions to model those from the actual problem that represents the situation accurately and enables them to relate the simplified problem to the actual problem. May be used in connection with, or in place of similar strategies; i.e., acting out the problem.)

Guess and check (Students shouldn't associate guessing for a solution with aimless casting about for an answer. The key element to this strategy is the "and check"; the problem solver checks his/her guesses against the problem conditions to determine how to improve his/her guess. The student repeats this process until the answer appears reasonable. This "guess and check" strategy gets the student involved in finding a solution by establishing a starting point from which he/she can progress. Used constructively with a table or graph this strategy may be a valuable tool.)

Work backwards (Frequently, problems are posed in which the final conditions of an action are given, and a condition that occurred earlier or that caused the final outcome is asked for. Under such circumstances, working backward may be valuable.)

Change your point of view (Some problems require that a different point of view be taken. A student with a "mind set" or certain perspective of the problem may have difficulty discovering a solution. Frequently, if the first plan adopted is not successful, the student returns to the point of view and adopts a new plan. This may be productive, but it might also result in continuous failure to obtain a solution. Students should try to discard previous notions of the problem and redefine the problem in a completely different way.)

Write an open sentence or equation (Often in conjunction with other strategies—using a table, diagram, etc., students select appropriate notation and attempt to represent a relationship between given and sought information in an open sentence.)

Carry Out the Plan

For some students, the strategies selected may not lend themselves to a solution. If a plan does not work, the problem solver should revise the plan or try another plan or combination of plans.

Look Back at the Steps Taken (Consolidate Gains)

Is the result reasonable and correct? Is there another method of solution? Is there another solution? Is obtaining the answer the end of the problem?

Generalize (Obtaining an answer is not necessarily the end of a problem. Reexamination of the problem, the result, and the way it was obtained, will frequently generate insights far more significant than the answer to the specific situation.

It may enable students to solve whole classes of similar and even more difficult problems.)

Check the solution (The very length of a problem or the fact that symbolic notation is used may make students lose sight of the original problem. Does the answer appear reasonable? Does it satisfy all the problem requirements?)

Find another way to solve it (Can you find a better way to confront and deal with the problem? The goal of problem-solving is to study the processes that lead to solutions. Having discovered a solution, students should search the problem for further insights and unsuspected ideas and relationships.)

Find another solution (Students approach many problems expecting only one correct solution. In many practical, daily-life situations, there may be many answers that are correct and acceptable.)

Study the solution process (Studying the process of solution makes problem-solving more than answer-getting and can expand an individual problem into a meaningful total view of a family of related problems.)

The four steps of the above model are not necessarily discrete. For example, students may move without notice into devising a plan while attempting to generate more information to understand the problem better.

If the four-step model is used, the key is to select an appropriate strategy or strategies to help answer the questions each step suggests. The strategies listed, and those students devise, should alter the problem information, organize it, expand it, and make it more easily understood. Strategies are the tools of problem-solving, and the four-step model, the blueprint.

The Broken-Stick Problem for Four Pieces — A Geometric Solution

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Consider the problem: A stick of length 1 is broken into four pieces. What is the probability that the four pieces can be joined to form a quadrilaterial? (Assume that the three points of breakage are independent random variables, each uniformly distributed on the interval [0, 1].

In this solution of the problem, the sample space is represented as a region in three-dimensional space.

Let the lengths of the four pieces be x, y, z and $1 - x - y - z$. Since each of these lengths is positive, the sample space may