It may enable students to solve whole classes of similar and even more difficult problems.)

Check the solution (The very length of a problem or the fact that symbolic notation is used may make students lose sight of the original problem. Does the answer appear reasonable? Does it satisfy all the problem requirements?)

Find another way to solve it (Can you find a better way to confront and deal with the problem? The goal of problem-solving is to study the processes that lead to solutions. Having discovered a solution, students should search the problem for further insights and unsuspected ideas and relationships.)

Find another solution (Students approach many problems expecting only one correct solution. In many practical, daily-life situations, there may be many answers that are correct and acceptable.)

Study the solution process (Studying the process of solution makes problem-solving more than answer-getting and can expand an individual problem into a meaningful total view of a family of related problems.)

The four steps of the above model are not necessarily discrete. For example, students may move without notice into devising a plan while attempting to generate more information to understand the problem better.

If the four-step model is used, the key is to select an appropriate strategy or strategies to help answer the questions each step suggests. The strategies listed, and those students devise, should alter the problem information, organize it, expand it, and make it more easily understood. Strategies are the tools of problem-solving, and the four-step model, the blueprint.

The Broken-Stick Problem for Four Pieces — A Geometric Solution

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Consider the problem: A stick of length 1 is broken into four pieces. What is the probability that the four pieces can be joined to form a quadrilaterial? (Assume that the three points of breakage are independent random variables, each uniformly distributed on the interval [0, 1].

In this solution of the problem, the sample space is represented as a region in three-dimensional space.

Let the lengths of the four pieces be x, y, z and $1 - x - y - z$. Since each of these lengths is positive, the sample space may be represented by the region S in three-dimensional space described by the inequalities $x > 0$, $y > 0$, $z > 0$ and $1 - x - y - z > 0$; that is, by the tetrahedron bounded by the co-ordinate planes and the plane $x + y + z = 1$. The volume of this tetrahedron is $1/6$.

The four pieces can be joined to form a quadrilateral if and only if the length of each piece is less than ½. Thus the set of selections favorable for the formation of a quadrilateral is represented by a region F consisting of those points that belong to S and whose co-ordinates satisfy the inequalities $x < ½$, $y < ½$, $z < ½$ and $1 - x - y - z < ½$.

The region F may be visualized as follows: Imagine the cube bounded by the co-ordinate planes and the planes $x = ½$, $y = ½$ and $z = ½$. F consists of the points inside this cube and between the parallel planes $x + y + z = ½$ and $x + y + z = 1$. The volume of the cube is $(½)^3 = 1/8$. The corner of the cube cut off by the plane $x + y + z = ½$ is a tetrahedron with volume $1/6\,(1/2)^3 = 1/48$ and the corner of the cube cut off by the plane $x + y + z = 1$ is also a tetrahedron with volume $1/48$. The volume of F is thus $1/8 - 1/48 - 1/48 = 1/12$.

The required probability is therefore given by:

$$\frac{\text{(The volume of F)}}{\text{(The volume of S)}} = \frac{(1/12)}{(1/6)} = ½.$$

For a solution of the Broken-Stick Problem for $n + 1$ pieces see: G.A. Heuer, "Solution to Problem E1480," *The American Mathematical Monthly,* 1962, 235-236.

A Mathematical Valentine

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One of the most familiar of the polar curves is the cardioid. The shape of the cardioid with equation $r = a\,(1 - \sin\Theta)$ is shown in Figure 1.

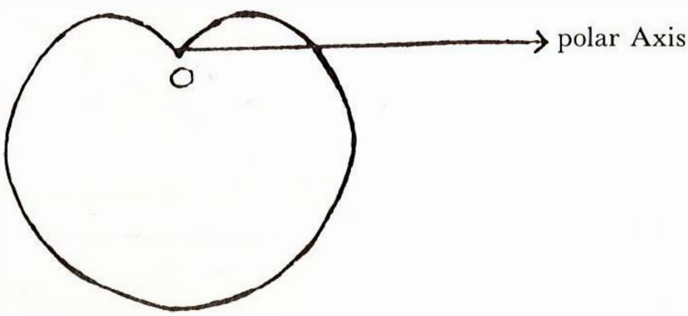

Figure 1