Preliminary Results from the Second International Mathematics Study in British Columbia

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During the 1980-81 school year, several thousand Grade 8 and Grade 12 mathematics students from British Columbia, along with their mathematics teachers, participated in the Second International Mathematics Study. More than 20 countries took part in various phases of the study, with Canada being represented through the participation of British Columbia and Ontario. A list of participants is given below:

Australia	Israel
Belgium (Flemish)	Ivory Coast
Belgium (French)	Japan
Canada (British Columbia)	Luxembourg
Canada (Ontario)	Netherlands
Chile	New Zealand
England	Nigeria
Finland	Scotland
France	Swaziland
Hong Kong	Sweden
Hungary	Thailand
Irish Republic	United States

The Second International Study of Mathematics is a project of the International Association for the Evaluation of Education Achievement (IEA). IEA is an association of educational research organizations and ministries of education whose primary goals are to conduct educational research on an international level and to assist member-states in undertaking co-operative research projects. IEA has conducted international surveys in the past, including the first mathematics study (Husén, 1967) and the Six-Subject Survey (Peaker, 1975; Walker, 1976). Twelve countries participated in the first mathematics study, but Canada was not among them.

The international study had a three-fold purpose:

- to investigate the mathematics being taught in those countries participating in the study.
- to compare the ways mathematics was being taught,
- to study the effects of that teaching on students' learning of and attitudes toward mathematics.

The data collected in the study will enable researchers to construct an international portrait of mathematics education.

Two populations of students were identified for investigation in the international study. The first, *Population A,* was defined as consisting of all those students enrolled in the grade where the majority of students have reached the age of 13 by the middle of the school year. In British Columbia, this corresponds to Grade 8. The international definition for the second group, *Population B,* was intended to encompass all students in the last year of secondary school who were studying mathematics as a significant part of an academic program. In other words, students who were studying college-preparatory mathematics in Grade 12 in British Columbia were to be included in Population B, but those who were not taking any mathematics or who were taking a terminal course were to be excluded. Population B was therefore defined to consist of all students in the province who were taking Algebra 12. It included approximately 40% of the Grade 12 population.

Because British Columbia was one of the first of the participants in the study to complete its data collection and to publish a local report (Robitaille, O'Shea, Dirks, 1982), few international results were available for comparison, except for a few concerning the curriculum. In addition, it has not yet been possible to examine the B.C. data for relationships between the strategies employed by teachers and the achievement levels attained by their students.

The following is a brief description of some of the major findings to date.

CURRICULUM ANALYSIS

• At the Population A (Grade 8) level, virtually all students in all of the participating countries are still studying mathematics. Moreover, the B.C. curriculum at that level coincides fairly well with the international grid developed for the study. In other words, the B.C. mathematics curriculum at the Grade 8 level is similar in many respects to the curricula of other countries.

• In some countries, only a very small proportion of the age group is still studying mathematics in the final year of secondary school; less than 10% in some cases. In British Columbia, for the Algebra 12 course, this figure is just over 30%. This fact has obvious implications for the content of the Algebra 12 course, since it must be made suitable for a comparatively large proportion of the student population.

• From the data obtained regarding the amount of time available in the school program for mathematics, it is clear that in many countries, students at the senior secondary level are able to specialize in mathematics to a much greater degree than is possible in British Columbia.

• A major difference between the Algebra 12 curriculum in British Columbia and the comparable curricula in other places is the importance given to the study of calculus. In B.C., calculus is not part of the Albegra 12 curriculum, except as one of a sequence of optional topics for the enriched format of the course. On the other hand, the study of calculus is a major component of the mathematics curriculum at the senior secondary level in other countries, notably in Europe.

• At both levels in British Columbia, little or no importance is given to the study of probability and statistics. This contrasts sharply with the importance given to these topics at either level or both levels in most other countries. More important, given the importance of these topics in contemporary society, this situation should be examined and discussed as soon as possible.

THE TEACHING OF MATHEMATICS IN GRADE 8

• Over 20% of Grade 8 teachers in B.C. teach students the British system of measurement as well as the metric system in spite of the facts that the Curriculum Guide calls for only the latter to be taught and that the prescribed textbooks refer only to the metric system. Teachers may have decided that this generation of students needs to be familiar with both systems, and, therefore, that both systems need to be taught. That the United States seems farther away from converting to the metric system than at any other time in the recent past lends additional weight to such an opinion, and this matter should be discussed as part of the curriculum renewal process.

• Teachers rarely assume that students have mastered concepts or skills in previous grades to the point where no reteaching or review is needed. Even topics such as concept of a fraction, which is introduced in the primary grades, are reported as having been taught or reviewed in almost 90% of the classrooms. This may be partly due to a certain lack of familiarity on the part of Grade 8 teachers with the content of the mathematics curriculum at the elementary school level. On the other hand, there is evidence that students perform relatively poorer on items dealing with content teachers have not reviewed.

• There are indications that in teaching computation techniques such as multiplication of decimals, teachers emphasize the sequence of steps to be followed in using the algorithm rather than an understanding of why the algorithm works the way it does. For example, 98% of teachers agree that they teach students how to multiply decimals by referring back to multiplication of whole numbers and giving students a rule for placing the decimal point in the answer. Similar, rule-oriented approaches to teaching are apparent for topics such as operations with positive and negative numbers, and the development of formulas.

• The amount of teaching time devoted to topics is highly variable. While every Grade 8 teacher spends some time teaching about integers, 10% say they spend, at most, eight class periods on the topic, while another 10% report devoting more than 20 periods to the same topic.

• In the opinion of teachers, the principal factors that account for students' lack of satisfactory academic progress in mathematics are students' indifference, absenteeism, and lack of ability. Few feel that unsatisfactory progress can be attributed to a shortage of appropriate resources for teaching or to a lack of teachers' expertise.

• According to teachers, whether or not a given approach to teaching a particular concept or skill is easy for students to understand is the single most influential factor in determining which of a number of alternative teaching approaches to use. This factor is substantially more important than whether or not students enjoy the approach or whether that approach is used in the textbook. That a given approach is seen as being difficult for students to understand is a powerful factor in decisions not to use a particular teaching approach.

• While teachers do not select teaching approaches primarily because they are present in the textbook, many are reluctant to follow approaches other than the ones in the textbook. This indicates that, if certain topics, approaches, or orientations are to be implemented in the mathematics curriculum, care must be taken to ensure that these are included either in the prescribed textbooks or in other instructional materials made available to teachers.

• The majority of teachers emphasize only one approach to teaching a given topic. While they might use others or refer to them occasionally, they tend to emphasize only one.

• Over 25% of Grade 8 teachers of mathematics report that, in their teaching, they seldom follow the treatment given to a topic in the textbook. Since a large number also report that they rely primarily on locally developed worksheets, many teachers may not be utilizing the prescribed textbooks to any significant degree.

• Although most Grade 8 teachers agree that students should be permitted to use calculators, they are also strongly of the opinion that students should master computational skills.

THE TEACHING OF MATHEMATICS IN GRADE 12

• There are very substantial differences in the amounts of time teachers spend on the different components of the Algebra 12 course. For analytic geometry, the range is from 6 to 45 periods, with 21 periods as the mean. In the case of trigonometry, the range is from 10 to 56 periods, with the mean being 29. For algebra, the range is from 7 to 96 periods, with the mean being 49.

• Approximately 40% of Algebra 12 teachers agree with a statement that there should be a separate, semester-long course in trigonometry. Only 3% disagree with this opinion; the remainder are undecided.

• As in the case of Grade 8, Grade 12 teachers spend substantial amounts of time reviewing material covered in earlier courses.

• Relatively little time is spent on developing students' ability to construct graphs of functions.

• Despite the fact that virtually all Algebra 12 teachers permit their students to use calculators, 50% of them emphasize the computational application of logarithms in their teaching.

• In presenting various formulas, identities, or generalizations to their students, almost all of the Algebra 12 teachers provide some form of proof or informal justification. However, they seldom expect their students to be able either to replicate such proofs or to produce them on their own. Instead, they expect students to recall the formulas and apply them.

• The use of calculators in class is permitted by 97% of Algebra 12 teachers. Moreover, 16% of the teachers also use microcomputers in some manner.

• Algebra 12 teachers adhere more closely to the course outline in the Curriculum Guide than do teachers of Mathematics 8, probably because the course outline for Algebra 12 is much more detailed than is the one for Mathematics 8.

• Algebra 12 teachers are more likely to use a particular teaching approach because they feel it will be easy for students to understand than because they think students will enjoy it. That a given approach is seen as being difficult is a stronger factor in deciding not to use a given approach to a topic than is the belief that students will not enjoy it.

• In a typical Algebra 12 class, correction of homework occupies 25% of the time, lecturing by the teacher an additional 40%, and individual seat work by students, 20%. Little time is allocated for having students work together in small groups.

• Results show that Algebra 12 teachers typically do not differentiate among levels of ability in their teaching. Most teachers teach the same content in the same way and give the same assignments to all students.

ACHIEVEMENT RESULTS GRADE 8

• Over 80% of the test items developed for use in the Second International Mathematics Study were judged to be appropriate for B.C. students in Grade 8. Students were told that the test contained some items with which they would not be familiar, and they were advised to skip those items.

• All Grade 8 students were administered a 40-item test at the beginning of the school year, and the test was re-administered near the end of the course. Results on the posttest were 4.4 points higher than those on the pretest, an improvement of about 25%.

• The range of growth scores on the 40-item core test was from − 1.8 to 10.1. That is, in one class the posttest score was 1.8 points lower than the pretest score, and, at the other extreme, the performance in one class improved by 10.1 points.

• Gains in achievement from pretest to posttest were greatest on the algebra and geometry subtests, the two areas that consisted primarily of content presented for the first time in Grade 8.

• In a number of cases, classes performed poorer on one or more of the subtests on the posttest than on the pretest. The losses occurred mainly on the subtests dealing with content from earlier grades; for example, fractions, measurement, and ratio and proportion.

• Teachers were asked to indicate whether or not the mathematics content required to respond to each item had been presented to students. Overall, there was only a moderate correlation between this Opportunity-to-Learn measure and the achievement results. This may mean that teachers underestimate their students' knowledge or that they are unfamiliar with the curriculum of earlier grades. Alternatively, it may be that teachers did not interpret the Opportunity-to-Learn question in the way that had been intended. Further analyses of these results should clarify this matter.

ACHIEVEMENT RESULTS GRADE 12

• No pretesting was done with Grade 12 students. Moreover, since the test items were distributed among eight rotated forms, there was no common set of items all students wrote.

• Of the total of 136 items on the eight test forms, 45 were judged to be inappropriate for B.C. students because they dealt with content outside the Algebra 12 curriculum.

• In constructing items for the tests, four levels of cognitive behavior were used: computation, comprehension, application, and analysis. In general, results show that achievement decreases as cognitive level increases. Similar results were found at the Grade 8 level.

• On the 91 items judged to be appropriate for the B.C. curriculum, the average percent correct was 42%. Given the level of difficulty of the items, this seems a satisfactory result.

STUDENTS' ATTITUDES

• When asked to react to a list of 15 topics and activities in mathematics, students at both the Grade 8 and Grade 12 levels indicated that they enjoy them, feel they are important, and do not find them too difficult.

• Teachers gave approximately the same ratings as their students to most of the items. However, teachers feel that problem-solving is more important, more enjoyable, and more difficult than their students do.

• Teachers and students agree that mathematics is a dynamic, growing field in which new developments are continually being made and not a static, unchanging one. Teachers are more positive in this regard than their students, and Grade 12 students are somewhat more positive than Grade 8 students.

• Students indicate that their parents consider mathematics to be an important area of study, and that their parents want them to do well in mathematics. Students at both levels report that, while both parents encourage them to do well in mathematics and believe that learning mathematics is important, their fathers enjoy mathematics more than their mothers and are more likely to be able to assist them with assignments.

• Overall, Algebra 12 students' attitudes toward mathematics are slightly more positive than those of Grade 8 students.

• A clear majority of students at both levels agree that it is important to know mathematics in order to get a good job; indeed, 32% of Grade 8 students strongly agree with that opinion.

• On a subscale consisting of four items, Grade 8 and Grade 12 students indicate that both males and females are equally capable in mathematics, have the same need for careers, and have the same need to know mathematics.

• Over two-thirds of students at both levels believe that it is important for them to know something about computers, and that computers can be used in beneficial ways.

The B.C. data have been submitted to the two international processing centres located at the University of Illinois and the New Zealand Department of Education. Most of the international analyses will be conducted at those two locations, although persons at a number of other European and North American institutions will also be involved in various aspects of the analyses as well as in the preparation of the international reports.

Three volumes are scheduled for publication over the next 15 months. The first will consist of a comparative analysis of the mathematics curricula in the participating countries. The second will deal with teaching practices and their relationship to student outcomes, and the third will deal primarily with achievement and attitudes. A conference to discuss the preliminary findings on an international level will be held in May 1983 at the University of British Columbia.

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