SQUARE ROOTS

ACTIVITY A

Do you remember the mathematics unit in which you learned how to find the area of a square figure? For example, what would be the area of a square tile with sides of length 12 inches?

e.g. χ represents length of a side of the square tile.

$$\frac{12}{1} \sim \frac{\chi}{12}$$

$$\chi = 12 \times 12$$

$$\chi = 144$$

The area of the square tile is 144 square inches.

Suppose we changed the problem so that you had to reverse the above operation to obtain the solution. That is, if given the area of a square, could you find the length of its sides?

For example, if you had a square with an area of 4 square inches, what would the length of its sides be?

____________ inches

Area =
4 sq. inches
or
$\chi^2 = 4$ sq. in.
$\chi = ?$

HINT: Find a number that, multiplied by itself, equals the area.

e.g. $\chi^2 = 4$

or $\chi \cdot \chi = 4$

Find χ and the problem is solved.

Ask your teacher for some square figures. You will notice that each has the area printed on it. Your job is to find the length of the side by <u>computation</u> - <u>rulers are a no-no!</u>

Use the hint given above.

Complete the table below for at least six of these squares:

(See next page)

	Area of Square	Length of Side (χ)	Check: Does Area of Square = χ^2
e.g.	4	2	$4 = 2^2$
a)			
b)			
c)			
d)			
e)			
f)			

Why don't you make up your own square, even if it's not a cut-out, and exchange your problem with a friend's and see if each of you can solve the length of the square's side.

ACTIVITY B

Congratulations!

By completing Activity A, in particular the second column, you have discovered a new operation called Finding the square root of a number.

This is the inverse operation of finding the square of a number, just as subtraction is the inverse of addition and division is the inverse of multiplication.

The symbol for the new operation is $\sqrt{\ }$

For example $\quad 5^2 = 25$

$$\sqrt{25} = 5$$

That is, the square root of 25 is 5.

How do you know? - $(5 \times 5 = 25)$

Compare the two operations again:

Find the square of 7: $\quad 7^2 = 7 \times 7 = 49$

Find the square root of 49: $\quad \sqrt{49} = \sqrt{7 \times 7} = 7$

Find the square root of the following numbers by completing the table:

	Number	Number Factored into EQUAL Factors	Square Root of Number ($\sqrt{\ }$)
e.g.	6^2	6 x 6	6
a)	64	8 x 8	
b)	9	-3 x -3	
c)	169	13 x 13	
d)	4/9	-2/3 x -2/3	
e)	625	25 x 25	
f)	225/121	15/11 x 15/11	
g)	64	-8 x -8	

Notice that a positive rational number may have both a:

1) positive square root, e.g., $\sqrt{64} = 8$.
2) negative square root, e.g., $\sqrt{64} = -8$.

How do we know which is correct?

Most mathematicians agree to use the square root symbol ($\sqrt{\ }$) to mean the positive square root only, or as it is also called, the principal square root. (See your text, p. 218.)

If the negative square root is wanted, then a negative sign is placed in front of the square root symbol.

Complete these: e.g. $-\sqrt{9} = -3$

$\sqrt{9} = 3$

$\sqrt{100} =$

$-\sqrt{100} =$

$-\sqrt{81} =$

$-\sqrt{1/4} =$

$\sqrt{121} =$

$-\sqrt{121} =$

Do you think it possible to find the square root of a negative number?

____

Why or why not? ______________________________

____

Before going any further, be sure you understand what the following terms mean:

SQUARE: a) the number obtained by multiplying two equal numbers or factors:

e.g., $2 \times 2 = 4$. 4 is the square of 2.

SQUARE: b) a plane geometric figure having four sides of equal length and each angle has a measure of 90 degrees.

SQUARING: a) the multiplication of a number by itself, or

b) the second power of a number.

SQUARE ROOT: one of the two equal factors of a number:

e.g., 3 is the square root of 9 because $3 \times 3 = 9$.

ACTIVITY C

USING A TABLE OF SQUARES AND SQUARE ROOTS

To save time, a table of squares and square roots is very helpful in finding these kinds of numbers. Turn to page 461 in STM 2 and study the table.

If you need help in answering the following exercises, turn to page 256 of your text.

Using the table on page 461, find the square and the square root of each of the following numbers:

(See next page)

	Number (χ)	Square (χ^2)	Square Root ($\sqrt{\chi}$)
e.g.	13	169	3.606
a)	38		
b)	49		
c)	4		
d)	212		
e)	68		
f)	5		
g)	137		

ACTIVITY D

The exact square root of 2?

The table on page 461 lists $\sqrt{2}$ as 1.414.

This should mean $(1.414)^2 = 2$. Does it? ____________________

$$1.414 \times 1.414 \text{ equals } 1.999396$$
$$\text{but } 1.999396 \neq 2$$

Obviously the book has made a mistake.

Try $(1.415)^2$. Does this equal 2? __________

What does it equal? __________

Try 1.4141 - __________

Try 1.4142 - __________

See if you can find the exact square root of 2.

ACTIVITY E

Calculating Square Root

Perhaps if we had a particular method for calculating the square root of a number, finding the square root of 2 might be easier.

Usually, mathematicians use a table of squares and square roots, or a slide rule, or a computer, but occasionally they might be stuck without any of these time-saving aids. Then they probably use one of the following methods.

1. GUESS METHOD*

Let's calculate the square root of 72. ($\sqrt{72}$)

First, find which integers surround $\sqrt{72}$

You know that $\sqrt{72}$ is greater/less than $\sqrt{64}$. $\sqrt{64} = 8$, therefore $\sqrt{72}$ is a number greater than 8. Is $\sqrt{72}$ a number greater/less than $\sqrt{81}$? $\sqrt{81} = 9$, therefore $\sqrt{72}$ is a number less than 9. This tells us that $\sqrt{72}$ is some number between 8 and 9.

Before we proceed with finding $\sqrt{72}$, you might find it helpful to practice the above procedure. Without using the table of squares and square roots, find the integers that bound the following numbers:

a) $\sqrt{72}$ lies between ______ and ______.

b) $\sqrt{75}$ lies between ______ and ______.

c) $\sqrt{26}$ lies between ______ and ______.

d) $\sqrt{13}$ lies between ______ and ______.

e) $\sqrt{95}$ lies between ______ and ______.

f) $\sqrt{135}$ lies between ______ and ______.

Returning to the calculation of $\sqrt{72}$, using the "guess method," we simply make successive guesses to continue our calculation.

The following table might help you make your calculation easier. Try and find $\sqrt{72}$ to the nearest thousandth.

(See next page)

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*Modified after D. C. Attridge, et al., ASTC Mathematics, (Toronto: GINN and Company, 1968), pp. 81-82.

	Guess	Guess "Squared"	Is Value Less than # (n), i.e., $\sqrt{72}$	Is Value Greater than #(n), i.e. $\sqrt{72}$	Range of Values for n, i.e. $\sqrt{72}$
i)	8	64	X		$8 < \sqrt{72}$
ii)	9	81		X	$8 < \sqrt{72} < 9$
iii)	8.5	72.25		X	$8 < \sqrt{72} < 8.5$
iv)	8.46	70.56	X		$8.46 < \sqrt{72} < 8.5$
v)					
vi)					
vii)					
viii)					

This method requires a lot of patience, but is easy to understand.

2. "NEWTON'S METHOD"

This is usually a shorter method and involves taking the average of two guesses. You will find the procedure detailed on pages 259-260 of your text.

3. ALGORITHMIC METHOD

This is also a good way of finding square root, although the method will likely make very little sense to you. Ask your teacher about this one if you don't like either of the first two methods.

Choose at least two of the following numbers and calculate the square root by whichever method you prefer (to the nearest thousandth).

a) $\sqrt{39}$ d) $\sqrt{155}$

b) $\sqrt{88}$ e) $\sqrt{19}$

c) $\sqrt{7}$ f) $\sqrt{140}$

Check your answers on page 461 of your text.

ACTIVITY F

One more square root calculation--maybe! If you have not already done so, from Activity D, calculate the $\sqrt{2}$ to five decimal places, using the method you prefer. Check your answer with your partner. Do you agree?

____

Does the number you have "squared" equal the numeral 2 <u>exactly</u>?

If not, how close are you? ____

Do you think you will ever find the exact square root of 2?

Why or why not? ____

____

If you did find it, or think you might find it, show your answer to the teacher, some mathematicians, your friends, parents, everyone, because you will be famous. You will have done what mathematicians have believed impossible for thousands of years.

BRAIN TEASER

What number multiplied by itself is equal to itself?

____

How many such numbers do you know? _____________________________________

Can you make any general multiplication statements about these numbers, i.e., statements that are true for any rational number:

____

____

TEACHERS' GUIDE

SQUARE ROOTS

LESSON OBJECTIVES:

1. Students learn the reverse operation of squaring a number--finding the square root of a number.
2. Students learn how to find square root by using a Table of Squares and Square Roots and/or one of several methods of calculation.

<u>Background Information</u>

ACTIVITY A

This is meant to be a discovery introduction to a simple calculation of square root. For classroom needs you will have to cut out a number of "perfect squares" e.g. squares of area 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, etc. To challenge some of your better students you might give them squares of mixed numerals or decimals e.g. - 6.25 - $(2\ 1/2)^2$, 19.36 - $(4.4)^2$, 11 1/9 - $(3\ 1/3)^2$, etc.

ACTIVITY B

The concept of the square root of a number being an <u>equal</u> factor is important (one of the two equal factors that form the number (product)).

e.g., $64 = 4 \times 16$ $\qquad 64 = 2 \times 4 \times 8$

$64 = 2 \times 32$ $\qquad 64 = 8 \times 8$

8 is the square root since it is one of an equal pair of factors that form the product. 4 is not a square of 64 since $4 \neq 16$.

In regard to finding the square root of a negative number, the answer is NO with respect to the numbers studied to date. The square root of a negative number is an imaginary number, but the decision is yours as to whether or not you want to discuss these at this time.

ACTIVITY C

Before or after completing this activity you might wish to have the students refer back to Activity F in the section on Squares and ask them to find the square roots of numbers given the square and using their graph. Perhaps you might introduce this exercise after Activity A in this section or at some other point, if at all, depending on your judgement.

ACTIVITY D

Unless you think the student is completely wasting his time, let him work on finding $\sqrt{2}$ as long as he is interested. There is a meaningful follow-up to this activity in the section on irrationals.

ACTIVITY E

1. Guess Method - basically an intuitive approach. Easy to understand but method can be long and laborious, particularly with larger numbers.

2. Newton's Method - while not labelled as such in STM 2 (probably because it is an adaptation of the original) this is a meaningful method.

3. Algorithmic Method - see page 169 STM 2 Teachers' Guide - probably the quickest way to calculate square root, but the most difficult technique to understand.

You may find the majority of students prefer the third method, after they have an understanding of the operation of square root, simply because it's quicker. If you think the student is spending too much time learning the different methods or making a choice, you might help him make a decision based on his ability and interest. In any case, no great stress need be laid on calculating square root, since 99.9 per cent of the students will probably use a Table of Squares and Square Roots if they encounter any future need for this operation.

ACTIVITY F

This is an open-ended activity intended as an introduction to the next section on irrational numbers.

Essential Activities

A, B, C, and D - these are the activities I think are most meaningful and beneficial to the majority of students. Activities E and F, dealing with calculation of square root simply involve a technique which we seldom use and quickly forget, but may be useful to the student in understanding square root and the existence of irrational numbers.

ANSWER KEY

SQUARE ROOTS

ACTIVITY A

$x^2 = 4$ sq. in. $x = 2$ length of side = 2 in.

Your answers will depend on the squares your teacher gave you.

ACTIVITY B (Square Root)

a) 8
b) -3
c) 13
d) -2/3
e) 25
f) 5/11
g) -8

Complete these:

$\sqrt{100} = 10$

$-\sqrt{100} = -10$

$-\sqrt{81} = -9$

$-\sqrt{1/4} = -1/2$

$\sqrt{121} = 11$

$-\sqrt{121} = -11$

The square root of a negative number is impossible to find considering your study of numbers to date.

The simplest explanation is based on the definition of square root, and from this we must conclude that there is no negative product formed by multiplying two equal numbers.

e.g., a positive number times a positive number = a positive number

a negative number times a negative number = a positive number

ACTIVITY C

	x^2	$\sqrt{x}$
a)	1444	6.164
b)	2401	7
c)	16	2
d)	44944	14.56
e)	4624	8.246
f)	25	2.236
g)	18769	11.705

ACTIVITY D

$(1.415)^2 = 2.002225$
$(1.4141)^2 = 1.99967881$
$(1.4142)^2 = 1.99996164$
$(1.4143)^2 = 2.00024449$

Good Luck!

ACTIVITY E

1. Guess Method

a)	8 and 9	d)	3 and 4
b)	8 and 9	e)	9 and 10
c)	5 and 6	f)	11 and 12

$\sqrt{72} = 8.485$ (to nearer thousandth)

3. Algorithmic Method

a)	6.245	d)	12.450
b)	9.381	e)	4.359
c)	2.646	f)	11.832

ACTIVITY F

$\sqrt{2} = 1.41421$

BRAIN TEASER

Two numbers:	0 and 1
For any a,	$a \times 0 = 0$
For any a,	$a \times 1 = a$

THE STORY OF JOE SINE

One day Joe Sine went to call on his new neighbors who lived in the adjacent house. He was a handsome tangent, a confirmed bachelor.

Joe was met at the door by two sisters who had anything but congruent figures. The first, Deca Gon, had real construction problems. Her discontinuous curves were intersected at various angles by parallel lines. The second sister, Polly Gon, was dressed in a pretty co-ordinate set and it was obvious that her natural curves ran into imaginary numbers. Just looking from the first to the second, Joe found his interest compounding rapidly. What poor Joe did not know was that Polly knew all the angles and was an expert at taking squares.

Joe was invited to come in and sit down. Deca proved to be as square as she looked and just sat there like a log. Polly, at a given sine, sent Deca out to find some roots to make tea. While she was gone, Polly served Joe pi. Then she used the complementary angle and Joe was soon reduced to zero power. Next she introduced an "If . . . then proposition." That is, if Joe would marry her, then . . . Joe said yes, then began to consider the possibility of spending the rest of his life adjacent to her side. He suddenly became very nervous and began crossing and uncrossing his legs for joint variation. What would be a Hero's formula for getting out of this mess? Joe was so scared he nearly had a corollary coronary.

But the tension was suddenly broken when Deca came bursting through the door with a tremendous discovery. Against all probability she had dug up a freak of nature, a square root!*

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*Mary Rector, "The Story of Joe Sine," In CHIPS from the Mathematical Log, ed. Josephine P. Andree (Norman, Oklahoma: The University of Oklahoma, 1966), p. 58.