STUDENT GUIDE

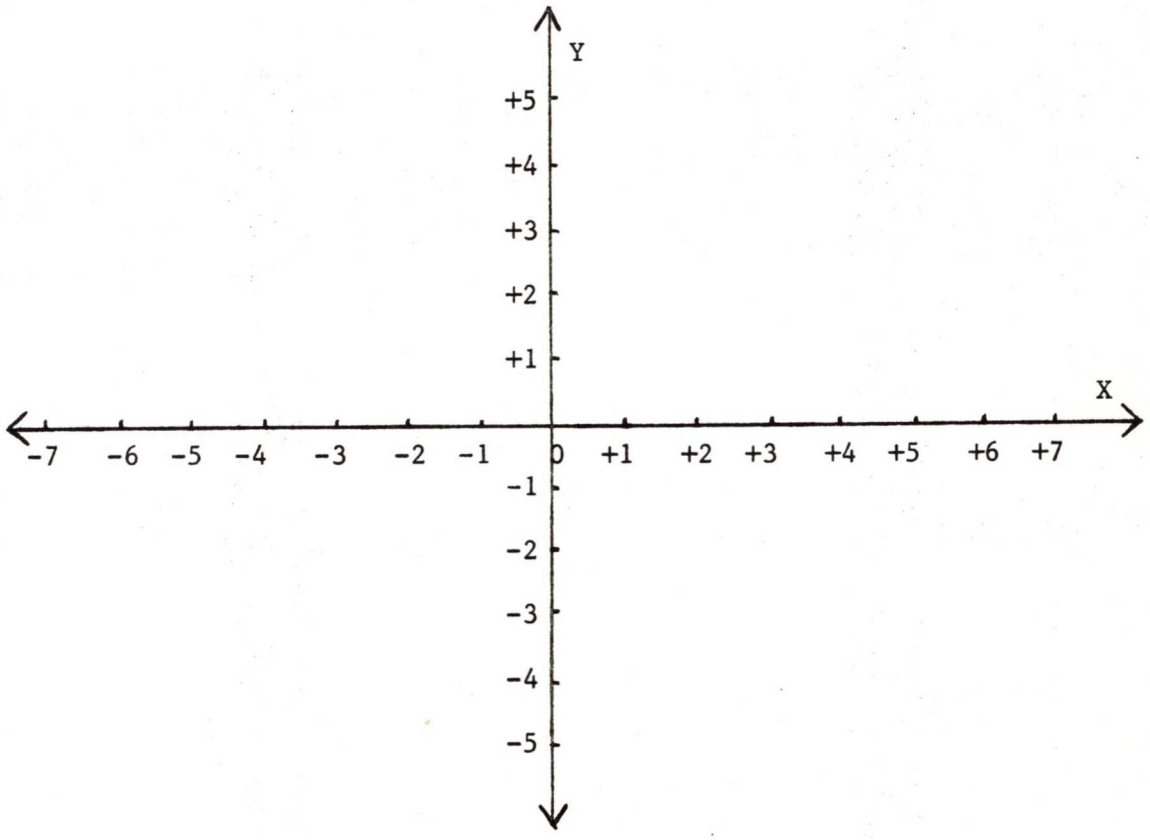
GRAPHING IN THE REAL NUMBER PLANE

In this particular section, you are going to see how certain mathematical procedures can be used to study the growth patterns of living organisms. An amazing discovery about living organisms (micro-organisms, plants, or animals--including humans) is that all have quite similar growth patterns. We shall investigate these growth patterns with a population of mold, but first we need to know some additional mathematical procedures.

ACTIVITY A

The Rectangular Coordinate System

There are a number of ways you can learn about associating ordered pairs of real numbers with the rectangular coordinate system (see the diagram below)



*1. You might ask your teacher to play the class game called "Living Coordinates." This is an activity in which every student in the class is identified by an ordered pair of numbers according to his position in the class seating plan. To be able to play you must learn how to "stay alive" by standing in response to certain questions the teacher will ask.

*2. You could play the game of "Tic-Tac-Toe," "Battleship," or "Go" on a grid similar to the one above and try and "discover" how to identify points on the grid with ordered pairs of real numbers. Since you might already know how to play one of these games, you will probably find this more fun and interesting. It will certainly be a challenge for you to solve this problem without the teacher telling you.

*3. Another method would be to study an explanation from a textbook or notes from your teacher about the rectangular coordinate system and complete any exercises that you might be assigned.

ACTIVITY B

Connect the Coordinates (dots)**

Graph each of the following points in the real coordinate plane

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*For more specific details on the activities named in suggestions 1 and 2, the reader is referred to the following teacher's guide on graphing in the real number plane. An excellent reference for suggestion 3 is: School Mathematics Study Group (SMSG), Mathematics and Living Things, Student Text, (Stanford, Calif.: Leland Stanford Junior University, 1965), pp. 129-137. The material in this text on these pages was actually duplicated and distributed with the original experimental unit. While the interested reader may wish to consult this reference to view the original context, he might be interested to know that the teachers thought this material presented too much of a conventional approach. Two of the teachers omitted the majority of this material and presented the students with the opportunity to discover these mathematical concepts on their own. They did this by trying to extend the game of "Tic-Tac-Toe" and "Battleship" to the complete rectangular coordinate grid from their previous experience with just the first quadrant. This method was very successful and well received by the students in the opinion of these two teachers. For this reason the investigator would suggest this approach in preference to the original material and suggestions for use.

**D. G. Seymour and R. Gidley, EUREKA (Palo Alto, Calif.: Creative Publications, 1968).

and connect the points in order with line segments. What do you see?

(-1, -2), (-6, -7), (-5, -7), (-4, -6), (-3, -7), (-2, -6),

(-1, -7), (0, -6), (1, -7), (2, -6), (3, -7), (4, -6), (5, -7),

(6, -7), (1, -2), (3, -2), (4, -1), (5, -2), (9, -2), (10, -1),

(11, -1), (12, 0), (13, 0), (14, 1), (15, 1), (16, 2),

(5, 2), (4, 1), (3, 2), (3, 4), (4, 5), (5, 5 1/2), (4, 6), (3, 6),

(2, 7), (0, 7), (-3, 4), (-3, 2), (-4, 1), (-5, 2), (-16, 2),

(-15, 1), (-14, 1), (-13, 0), (-12, 0), (-11, -1), (-10, -1),

(-9, -2), (-5, -2), (-4, -1), (-3, -2), (-1, -2).

ACTIVITY C

The World's Most Famous Beagle!

I. Plot the following points and join them in order with straight lines:

(-8, 2), (-6, -2), (-6, -6), (-8, -8), (-12, -6), (-12, 0),

(-8, 8), (-4, 9), (-2, 8), (0, 4), (4, 4), (7, 1), (7, -1),

(2, -4), (-2, -4), (-2, -8), (0, -10), (-6, -10), (-4, -8),

(-4, -4), (-6, -4). Join these points (-3, -3), (-2, -2),

(-2, 4).

Make a big dot ● at the points (-4, 4) and (-2, 4).

Make an oblong circle around the X axis between 2 and 4.

Shade the circle in.

Do you recognize this World War I flying ace?

Make a big dot at the points (-4, 4) and (-2, 4).

Make an oblong circle around the X axis between 2 and 4.

Shade the circle in.

Do you recognize this World War I flying ace?

What's his name?

What is the name of the comic strip he appears in?

To make him look more authentic, shade in his ear.

II. Take the above ordered pairs and divide each value by 2. This will give you the points:

(-4, -1), (-3, -1), (-3, -3), (-4, -4), (-6, -3) etc.

Complete the rest of these and join them up as before. What has happened to Snoopy? _____

_____

III. Using the original ordered pairs, keep the X-value the same and double the Y-value. This will give you the points:

(-8, 4), (-6, -4), (-6, -12) etc.

Complete these, plot them and join them up. Describe your dog now. _____

_____

IV. Using the original ordered pairs, keep the Y-value the same and double the X-value. Plot these, join them up and describe Snoopy now. _____

_____

V. This time take the original values again, leave the Y-value alone and add 1 to every value of X. What's happened to Snoopy? _____

_____

Experiment with some other techniques of changing the X and Y values and see what happens to Snoopy. Compare them or try them out on your partner.

ACTIVITY D

The following activity is a more challenging one for those of you who enjoy plotting animal figures. See what kind of animal you end up

with this time. Any guesses!

NOTE that the instructions are very similar except that the ordered pairs of numbers are in table form.

What about plotting Charlie Brown, Woody Woodpecker, Yogi Bear, Robin Hood? Get a coloring book or some picture of your favorite cartoon character, or any other character or figure you prefer (not too detailed). Trace their picture on graph paper and then map out a set of ordered pairs that when joined up will show your hero. Try it out on your partner. Have fun.

ACTIVITY E

Tic-Tac-Toe

Perhaps you have played this game using coordinates in the first quadrant. Have you ever played X's and O's? This game can be played with many variations of the rules. To play you simply need a graph of the real plane and two players or two teams. Each team takes turns calling ordered pairs to be marked on the graph. The object of the game is to get 4 of your points on the graph that can be joined in a straight line. The winning player or team is the one with the most straight lines (or points if you award a point for each line).

After playing this one or more times, can you think of some new rules or changes in the old rules to make the game different and/or more interesting. Call it your version of Tic-Tac-Toe.

If you like these kinds of games ask your teacher about:

1. "Battleship."* Each team tries to sink the other team's "Navy" by firing shots of ordered pairs onto the enemy battlefield (graph).

2. "The Point-Set Game."** A version of the ancient Japanese war game called "GO." See who will be the first to holler "atari." A very interesting and challenging game.

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*Edith E. Biggs, and James Maclean, Freedom to Learn (Don Mills, Ont.: Addison-Wesley (Canada) Ltd., 1969), p. 51.

**Robert, B. Davis, Discovery in Mathematics. A Text for Teachers (Reading, Mass.: Addison Wesley Publishing Co., 1964), pp. 55-66.

GRAPHING PICTURE*

	X	Y
	-2	3
*	-3	4
	-4	3
	-2	3
	-2	-3
	2	-3
	2	3
*	3	4
	4	3
	2	3
	lift pencil	
	-2	-3
	-1½	-4
*	1½	-4
	2	-3
	-2	-3
	lift pencil	
	-5½	-2½
	-6	-3
	-6	-1½
	-8	-2
	-6	½
	-8	1
	-6	2
	-9	5
	-6	4
	-6	5
	-4	3½
	-5	7
	-2	5
	-2	9
	0	5
	2	9
	2	5
	5	7
	4	3½
	6	5
	6	4
	9	5
	6	2
	8	1
	6	½

X	Y
8	-2
6	-1½
6	-3
5½	-2½
lift pencil	
-4	-2
-5	-3
-5	-5
-9	-7
-7	-4
-9	-5
-9	-3½
-12	-3
-11	½
-12	1
-11	3½
-12	5
-11	5
-11½	7½
-11	8
-12	11
-9	10
-10½	14
-7	12
-7	13
-6	12
-6	14
-5	12
lift pencil	
-6	9½
-5½	12
-4½	11
-4	14
-2½	12
-2	13
-½	12
0	12½
½	12
2	13
2½	12
4	14
4½	11

X	Y
5½	12
6	9½
lift pencil	
5	12
6	14
6	12
7	13
7	12
10½	14
9	10
12	11
11	8
11½	7½
11	5
12	5
11	3½
12	1
11	½
12	-3
9	-3½
9	-5
7	-4
9	-7
5	-5
lift pencil	
4	-2
5	-3
5	-6
3	-8
2	-8
1	-9
-1	-9
-2	-8
-3	-8
-5	-6
-5	-5
lift pencil	
-4	-5
-2½	-6
2½	-6
4	-5

*Robert C. Madison, Graphing Pictures (Des Moines, Iowa: Central Iowa Mathematics Project (CILAMP), 1969).

ACTIVITY F

Growing Mold

The experience and knowledge you have gained from the previous activities will be used in this biological experiment. The experiment is based on a study of the growth pattern of a mold culture and will involve the ability to read a rectangular coordinate graph to identify the position of the mold.

Materials and Supplies

You will need an aluminum pie or cake tin, 10 × 10 to the inch graph paper, gelatin, a bouillon cube, Saran wrap, a rubber band or Scotch tape, scissors and a ruler for each group.

Procedure

1. To prepare the tin, cut the graph paper to fit into the bottom of the tin. The X and Y axes must be constructed on the graph paper and then rubber cement or glue should be used to hold the graph paper to the bottom of the tin.
2. To prepare the gelatin mix, combine a cold mix of gelatin with a hot mix of a bouillon cube and be ready to pour before the mix sets.
3. Pour a thin layer of the mix onto the graph paper in the bottom of the tin. Put the tin aside to cool and set for approximately 5 minutes. While it is "setting," it will be contaminated with mold from the air.
4. Cover the top of the tin with an excess of Saran wrap or a similar transparent wrap and fasten with a rubber band or Scotch tape.
5. Store in a dark place where the temperature is fairly uniform.
6. Observe and record your observations every day.

Recording

The mold will probably become visible on the 2nd, 3rd, or 4th day and as soon as it does, the following method should be used.

1. Identify the position of each dot of the mold on the coordinate grid at the bottom of the tin.
2. Transfer the position of each dot of mold to a separate piece of graph paper according to both its position and the number of squares covered. This procedure is to be completed every day.
3. The data should be used to complete the following table (see Table M).
4. From the total growth data in the table a graph should be plotted. Use the day number for the x-axis and the y-axis for area of squares covered. (See Figure A on the second page following.)

TABLE M*

RECORD OF MOLD GROWTH

A	B	C	
DAYS	TOTAL AREA TO DATE (SQUARE UNITS)	INC. AREA OVER PREVIOUS DAY (SQUARE UNITS)	PERCENT OF INCREASE- $\frac{\text{INC. AREA}}{\text{TOTAL AREA}}$
Start (Friday)			
2nd. Saturday 3rd. Sunday			
4th. Monday 5th. Tuesday 6th. Wednesday 7th. Thursday 8th. Friday			
9th. Saturday 10th. Sunday			
11th. Monday 12th. Tuesday You may go on . .	.		

*Modified after Mathematics for Living Things, Student Text, rev. ed. (Stanford, Calif.: Leland Stanford Junior University 1965), p. 141.

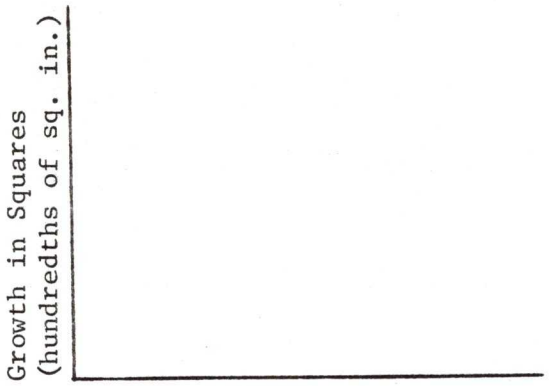


FIGURE A

5. Upon completing the experimental procedures, analyse your results by comparing them with a "typical" growth curve and the growth curves obtained from other living organisms. (See your teacher for this information.)

NOTE: The detailed instructions necessary for this student experiment are found on pages 138-143 of Mathematics for Living Things, Student Text, SMSG (Stanford, Calif.: Leland Stanford Junior University, 1965). These pages were duplicated for the students in the original unit and the reader will find complete information for the student experiment from this source. For additional information see Activity F in the Teacher's Guide of this section of the Appendix.

TEACHER'S GUIDE

GRAPHING IN THE REAL PLANE

LESSON OBJECTIVES:

Students should learn how to associate ordered pairs of real numbers with points in the real plane.

ACTIVITY A

The purpose of the activities mentioned in the student guide is to teach the student the mathematical procedures involved in identifying points in the real plane. In using these activities, teachers should select the most appropriate one based on the abilities and interests of the students. (See the explanatory note at the end of Activity A, student guide.)

1. "Living Coordinates"
   This is an excellent introductory activity although the students should have some preliminary introduction to the rectangular coordinate system. For full details the reader is referred to pages 109-112 of Mathematics for Living Things, Teachers' Commentary, rev. ed., School Mathematics Study Group (Stanford, Calif.: Leland Stanford Junior University, 1965). A summary description is also included in Chapter 4 of this study.
2. "Battleship"
   This game is described on page 51 of Freedom to Learn, E. E. Biggs and J. R. Maclean (Don Mills, Ont.: Addison-Wesley (Canada) Ltd., 1969). Please note that this game can easily be modified to correspond with the identification of points on the grid according to the intersection of two lines.
   "Go"
   A version of this game is described under the heading "The Point Set Game" by Robert B. Davis in the text Discovery in Mathematics, A Text for Teachers (Reading, Mass.: Addison Wesley Publishing Co., 1964).
3. As mentioned previously in the student guide the reader is referred to a detailed and thorough development of the rectangular coordinate system along with various exercises in pages 129-137 of School Mathematics Study Group of Mathematics for Living Things, Student Text, rev. ed. (Stanford, Calif.: Leland Stanford Junior University, 1965). There is also a published teacher's guide and answer key for these activities found on pages 102-109 of School Mathematics Study Group of

Mathematics for Living Things, Teachers' Commentary, rev. ed. (Stanford, Calif.: Leland Stanford Junior University, School Mathematics Study Group, 1965). The teacher's guide in the original experimental unit included duplicated copies of these pages.

ACTIVITIES B, C, and D

These are puzzle and enrichment activities that students can work on according to their interests. They should prove highly motivating to most students and if additional activities of this nature are desired, the manual Graphing Pictures by Robert C. Madison provides a rich source of materials. However, before supplying the students with many more plotting activities of this type, you might encourage them to make up their own with the aid of pictures or a coloring book. An example of the caricature of "Leo the Lion" which would result from the instructions given in Activity D is included for the teacher's use.

ACTIVITY F

Growing Mold

This is the major activity of this section and is an excellent example of active learning. Not only does it provide the student with "concrete" experience and an opportunity to use his mathematical skills gained from the previous unit, but it offers the teacher some excellent opportunities to coordinate mathematics with the subject areas of science and social studies.

Background Information

Growth is a major biological concept and if it can be measured early enough and late enough in the life of the organ (such as heart, liver, brain, etc.), organism (total living thing), or population, then three phases should be evident in the final graph:

*1. The lag phase: illustrating a slow start, a period of cell or organism adjustment.

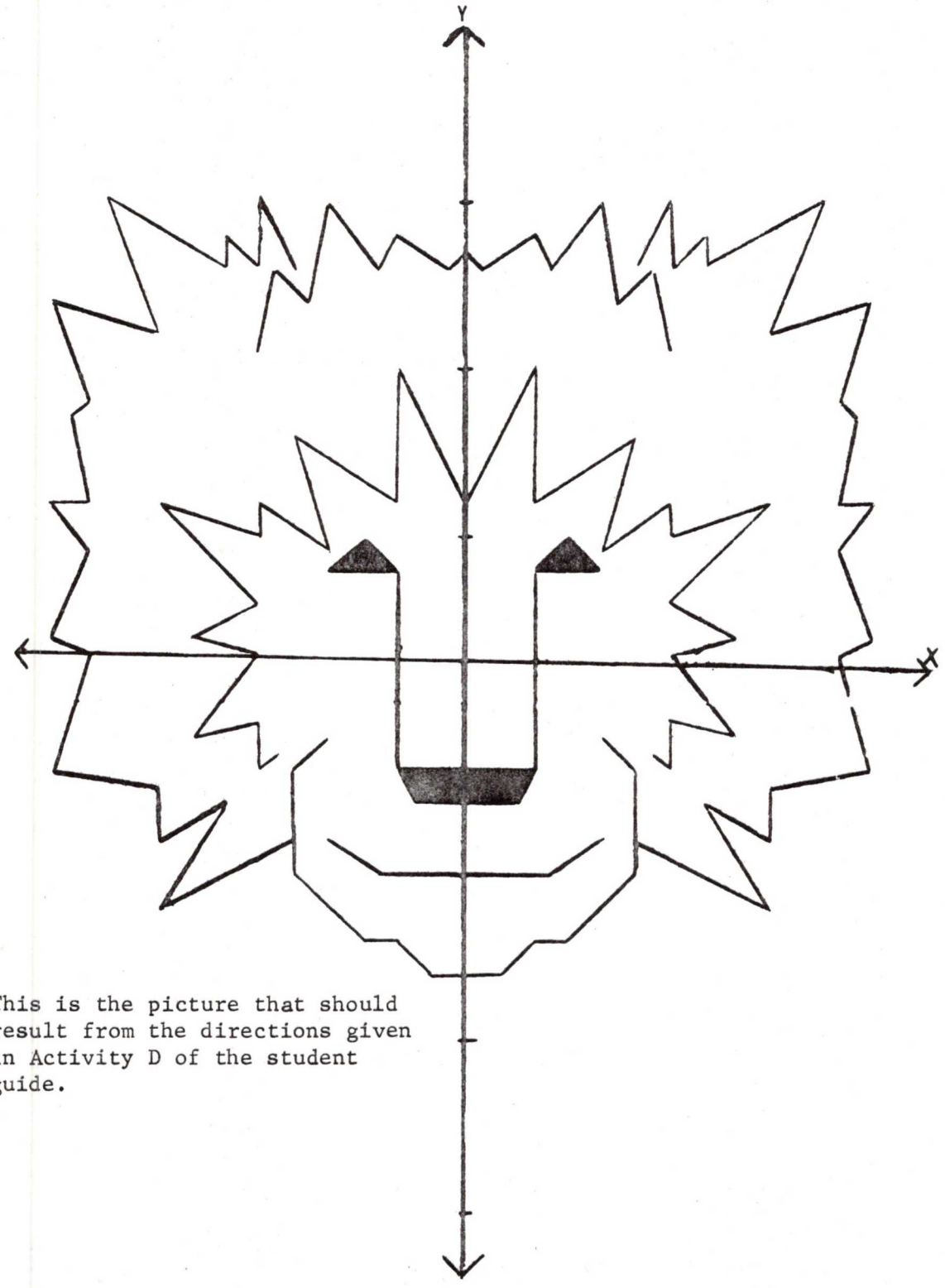
*2. The grand phase or exponential phase, where the cells are multiplying exponentially (1 cell divides into 2, the 2 divide into 4-8-16-32-64- etc., becoming astronomical in number before limiting factors begin to be felt), and

*3. The stationary phase or senescence, a levelling off as a result of a completed set of limiting factors, such as

(continued on third page following)

*School Mathematics Study Group, Mathematics for Living Things, Teachers' Commentary, rev. ed. (Stanford, Calif.: Leland Stanford Junior University, 1965), p. 100.

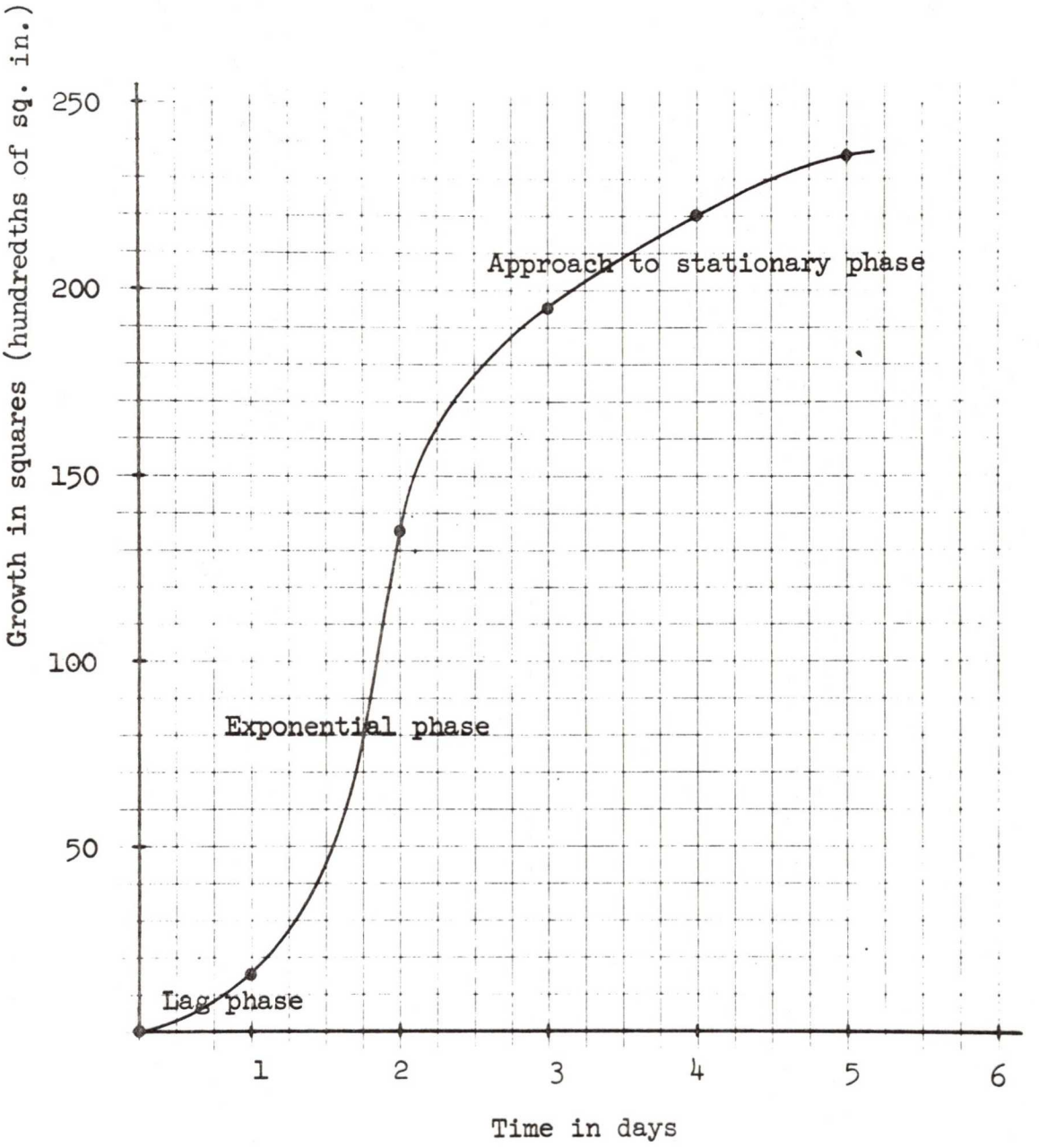
THE LION



This is the picture that should result from the directions given in Activity D of the student guide.

Source: Robert C. Madison, Graphing Pictures (Des Moines, Iowa: Central Iowa Mathematics Project (CILAMP), 1969).

The following graph shows the results obtained during five days of growth in an activity carried out according to the instructions in the student text.



School Mathematics Study Group (SMSG), Mathematics for Living Things, Teachers' Commentary (Stanford, Calif.: Leland Stanford Junior University, 1965), p. 113.

metabolism, regulation, food supply, and relationships of the organisms with each other and their environment.

Materials, Recording and Procedure

In addition to the details and reference made under Activity F of the Student Guide of this section the following pages should be of assistance to the teacher: Pages 100-101 and pages 112-120 of Mathematics for Living Things, Teacher's Commentary, rev. ed. School Mathematics Study Group (Stanford, Calif.: Leland Stanford Junior University, 1965). The majority of these pages were duplicated for teacher reference and use in the original unit.

Upon completion of the experiment on growing mold, a further study of the biological process of growth is encouraged by plotting and studying the growth curves of other living organisms such as gourd fruit, bacteria, chickens, corn plants, the liver and brain of human boys and girls as well as the population growth curves of the U.S. and Canada. The study and analyses of these growth curves contributes not only to an understanding of the biological phenomena of growth but also provides an opportunity to discuss and study the problems of population explosion, birth control and pollution.

CANADA'S GROWTH CURVE

The final activity on plotting growth curves is not only an excellent summary but a superb opportunity to introduce such terms as "population explosion," birth control, pollution, population control, etc. Population statistics for Canada and the world are listed. Population figures for Canada were not officially taken until 1851 and 1969 is the most recent. The growth curves that result from the data are excellent for comparing with the standard growth curve on page 162.

OFFICIAL POPULATION STATISTICS FOR:

A. Canada

Year	Population
1851	2,436,000
1861	3,230,000
1870	3,625,000
1880	4,225,000
1890	4,779,000
1900	5,301,000
1910	6,998,000
1920	8,556,000
1930	10,208,000
1940	11,381,000
1950	13,712,000
1960	17,870,000
1969	21,066,000
1980	?
1990	?
2000	?

B. World

Year	Population (Millions)
1650	545
1750	728
1800	906
1850	1,171
1900	1,608
1930	2,070
1940	2,295
1950	2,517
1960	3,005
1967	3,402
1980	?
1990	?
2000	?