STUDENT ACTIVITIES

SQUARES

ACTIVITY A*

Continue the pattern in this table:

SHAPE	.	. . . .	. . . .	. . . .		
NUMBER OF DOTS	1	4				

What shape have you drawn? _____

These numbers are called <u>square numbers</u>.

ACTIVITY B*

Here is another way to find square numbers -

Finish the chart:

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*Ray W. Cleveland, unpublished manuscript, 1968.

$1 = 1$

$1 + 3 = 4$

$1 + 3 + 5 =$ _____

$1 + 3 + 5 + 7 =$ _____

$1 + 3 + 5 + 7 +$ _____ $=$ _____

$1 + 3 + 5 + 7 +$ _____ $+$ _____ $=$ _____

$1 + 3 + 5 + 7 +$ _____ $+$ _____ $+$ _____ $=$ _____ .

What kind of numbers did you add to get square numbers?

_____

ACTIVITY C

Another way to find square numbers is to multiply each number by itself:

$1 \times 1 = 1$

$2 \times 2 = 4$

$3 \times 3 =$

$4 \times 4 =$

$5 \times 5 =$

The product 3×3 may be written as 3^2.

The number 2 is called an <u>exponent</u> and 3 is called the <u>base</u>.

Other examples are: $4 \times 4 = 4^2$

$7 \times 7 = 7^2$

$2/3 \times 2/3 = (2/3)^2$

$1\ 5/8 \times 1\ 5/8 = (1\ 5/8)^2$

ACTIVITY D

Compute the value for each of the following:

(a) $8^2 = 8 \times 8 = 64$

(b) $5^2 = 5 \times 5 =$

(c) $0^2 =$

(d) $(1/3)^2 =$

(e) 12^2 =

(f) $(-3/4)^2$ =

(g) 9^2 =

(h) $(8/9)^2$ =

(i) 12^2 =

(j) $(11/13)^2$ =

(k) $(-5/8)^2$ =

(l) $(.5)^2$ =

(m) $(2.2)^2$ =

Multiplying 4 by 4 and obtaining the result, 16, is called "squaring 4." In other words, 4^2 can be read "4 squared."

Can you think of another way of defining the "square" of a number?

From the examples above, you should agree that the square of a number is the second power of a number.

36 is the square of 6. ($6^2 = 36$)

9/25 is the square of 3/5. ($(3/5)^2 = 9/25$)

As you can see, we may "square" any rational number.

But when we square a subset of the rational numbers, the positive integers, I_p, the squares of these numbers are called perfect squares or square numbers.

Look back at Activity D. Which of the numbers that you squared are perfect squares?

Are the square numbers that you worked with in Activities A, B, and C perfect squares?

ACTIVITY E

Squaring members of the set R_N.*

Square the following numbers:

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*Modified after van Engen et al., Seeing Through Mathematics, special edition, Book 2, Toronto: W. S. Gage Limited, 1964.

$(-5)^2 = -5 \times -5 =$

$(-4/7)^2 =$

$(-10)^2 =$

$(-1\ 2/5)^2 =$

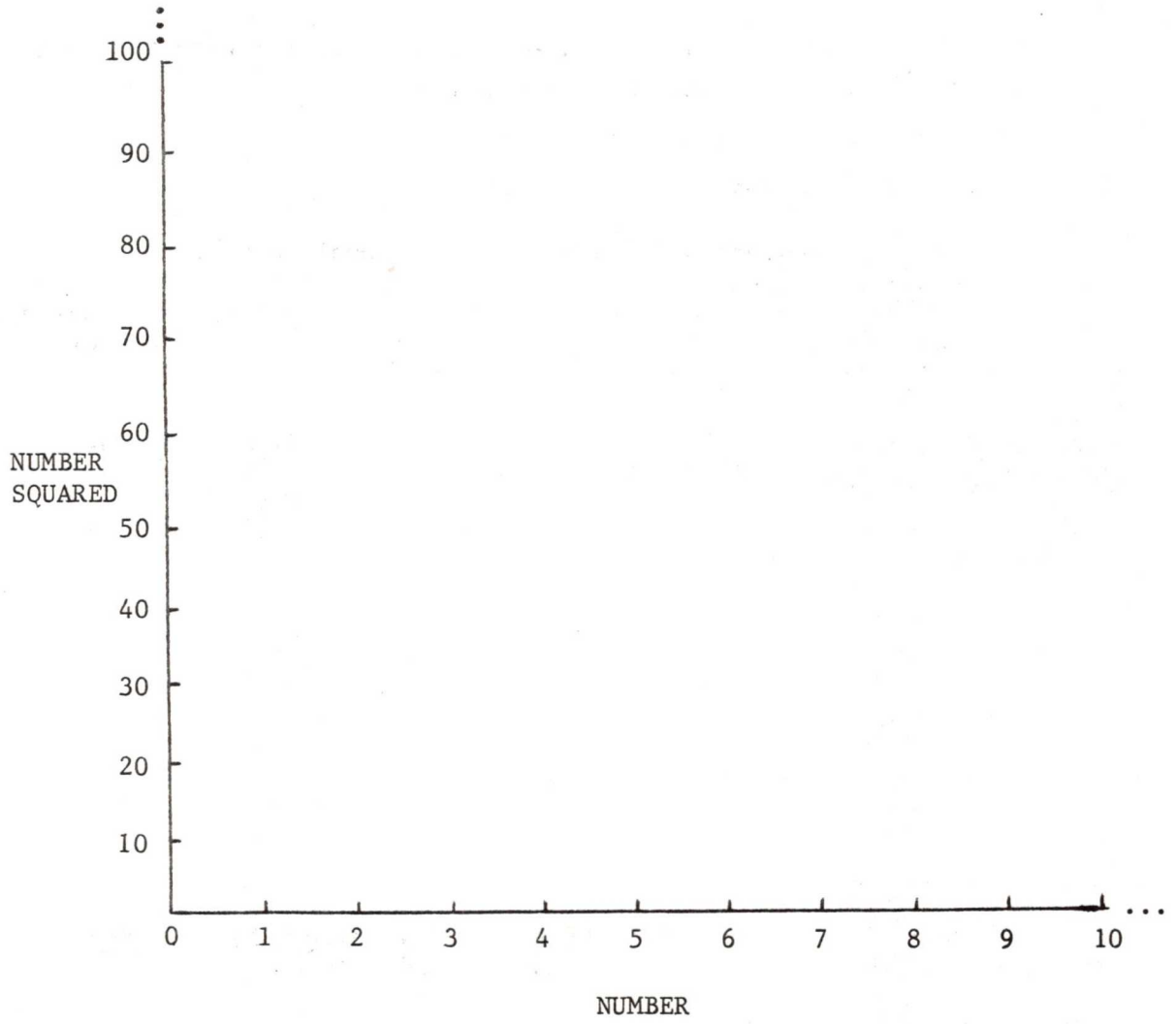
$(-3.3)^2 =$

$(-7)^2 =$

What do you notice about the square of a negative number?

ACTIVITY F

Tabulate the ordered pairs for the integers from 1 to 10 and the squares of these integers. Use the ordered pairs to complete the graph below:



Beginning at the origin, join these ordered pairs of numbers with one continuous line.

Examine your graph. Can you use this graph to find the approximate squares of other numbers?

Using your graph only, try and find the square of

(a) 2 1/2

(b) 7 1/2

(c) 4 3/4

Check your answers with your partner and see how closely you agree.

ACTIVITY G

Following is a list of rational numbers and their squares.

Try and pair them up.

Example: $(-6)^2 = 36$

$(-6)^2$, 4^2, 28 57/64, 4225, $(-5\ 3/8)^2$, $(-11)^2$, 16, 6.5^2, 65^2, 20^2, $3/8^2$, 121, $(7/8)^2$, 36, 49/64, 42.25, 400, 9/64

If you wish, make up your own mixture of numbers and their squares and see if your partner can match them.

ACTIVITY H

Try and express every integer from 1 to 25 as the sum of not more than four square numbers. Can it be done?

1 = 1	10 =	19 =
2 = 1 + 1	11 =	20 =
3 =	12 =	21 =
4 =	13 =	22 =
5 =	14 = 9 + 4 + 1	23 =
6 =	15 =	24 =
7 =	16 =	25 =
8 =	17 =	
9 =	18 =	

*If you have time, ask your teacher or check some mathematics books for information on how to find: triangular numbers; pentagonal numbers.

TEACHERS' GUIDE

SQUARES

LESSON OBJECTIVES:

1. Students learn how to find the square of a rational number.
2. Students learn that a perfect square or square number is the square of a positive integer.

Essential Activities

1. Activities D, E, and G - a bare minimum.
2. Remaining activities may be considered optional depending on student ability, interest and class time.
3. Strongly recommend activities A, B, C plus F and H, since they are not difficult and should prove interesting to students.

Additional Information

Activity F

Instructions are not too clear, I believe. Make it plain in tabulating the ordered pairs that:

(a) 1st component of ordered pairs are the numbers from 1 - 10.

(b) 2nd component of ordered pairs is the square of the 1st component, (e.g., (5, 25).

For the student graph, the use of graph paper should result in a much more accurate graph line as well as making the task of reading the graph to find squares easier and more accurate. Students should save this graph for a subsequent activity in the section on square roots.

Activity H

Triangular Numbers

SHAPE	.	∴		
NUMBER OF DOTS	1	3	?	?

Pentagonal Numbers

SHAPE	.	⁘		
NUMBER OF DOTS	1	5	?	?

NOTE: Pentagonal numbers are the sum of triangular numbers and square numbers i.e. 5 = 4 + 1. Students might try adding some more numbers and shapes to each series.*

*Modified after M. A. Hervey, and B. H. Litwiller,"Polygonal numbers: a study of pattern," The Arithmetic Teacher, Vol. 17, No. 1 (January, 1970), pp. 33-38.

ANSWER KEY

SQUARES

ACTIVITY A

Number of Dots: 1 4 9 16 25 36

What shape have you drawn? Square

ACTIVITY B

$1 + 3 + 5 = 9$

$1 + 3 + 5 + 7 = 16$

$1 + 3 + 5 + 7 + 9 = 25$

$1 + 3 + 5 + 7 + 9 + 11 = 36$

$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$

To get square numbers we added consecutive ODD NUMBERS

ACTIVITY C

$3 \times 3 = 9$

$4 \times 4 = 16$

$5 \times 5 = 25$

ACTIVITY D

(a) 64
(b) 25
(c) 0
(d) 1/9
(e) 144
(f) 9/16
(g) 81
(h) 64/81
(i) 144
(j) 121/169
(k) 25/64
(l) .25
(m) 4.84

Which are perfect squares? a, b, e, g, i

The numbers 1 4 9 16 25 36 49.

ACTIVITY E

$(-5)^2 = 25$

$(-4/7)^2 = 16/49$

$(-10)^2 = 100$

$(-1\ 2/5)^2 = 1\ 24/25$

$(-3.3)^2 = 10.89$

$(-7)^2 = 49$

The square of a negative number is a positive number or:
A negative number times a negative number equals a positive number. (or any equivalent answer)

ACTIVITY F

Ordered Pairs:

(1,1)
(2,4)
(3,9)
(4,16)
(5,25)
(6,36)
(7,49)
(8,64)
(9,81)
(10,100)

Using graph to find squares of:

(a) 2 1/2 - 6 1/4
(b) 7 1/2 - 56 1/4
(c) 4 3/4 - 22 9/16

Approximate answers are satisfactory

ACTIVITY G

$(-6)^2 = 36$

$4^2 = 16$

$(-11)^2 = 121$

$(-5\ 3/8)^2 = 28\ 57/64$

$65^2 = 4,225$

$20^2 = 400$

$(3/8)^2 = 9/64$

$(6.5)^2 = 42.25$

$(7/8)^2 = 49/64$

ACTIVITY H

1 = 1
2 = 1 + 1
3 = 1 + 1 + 1
4 = 1 + 1 + 1 + 1
5 = 4 + 1
6 = 4 + 1 + 1
7 = 4 + 1 + 1 + 1
8 = 4 + 4
9 = 4 + 4 + 1
10 = 9 + 1
11 = 9 + 1 + 1
12 = 9 + 1 + 1 + 1
13 = 9 + 4
14 = 9 + 4 + 1
15 = 9 + 4 + 1 + 1
16 = 4 + 4 + 4 + 4
17 = 16 + 1
18 = 9 + 9
19 = 9 + 9 + 1
20 = 16 + 4
21 = 16 + 4 + 1
22 = 9 + 9 + 4
23 = 9 + 9 + 4 + 1
24 = 16 + 4 + 4
25 = 16 + 9

NOTE: Answers are not unique - other sums possible

e.g. 17 = 16 + 1

17 = 9 + 4 + 4