Mathematics Council
NEWSLETTER
The Alberta Teachers' Association

Volume 9 Number 4 April 1991

From the Editor

The Curriculum and Evaluation Standards for School Mathematics has been available for two years. This is likely the most profound document ever published outlining the direction for mathematics education as we move into the twenty-first century.

Is the information in this document just gathering dust on some shelf, or are the recommendations being implemented into the mathematics education programs of schools in Alberta?

The following article, taken from The National Council of Supervisors of Mathematics Newsletter (January 1991), provides food for thought.

The Principal's Guide to the Implementation of the Standards (Or, How One Can Tell if the Standards Are Being Implemented in the Classroom)

by Harry Stratigos

General Perceptions/Beliefs

The teacher believes and promotes the ideas that

* math is valuable for survival and for better understanding of the world;

* math is for everyone;

* math is connected to other instructional areas and to the real world;

* problem solving is the primary purpose of math.

Observable Behaviors

The teacher

* gives the students assignments that contain fewer computational exercises and more problems requiring higher order thinking processes;

* allows students to use calculators as needed;

* has stopped using trig and log tables and interpolation, and requires students to use scientific calculators;

* introduces problems involving patterns, relationships and geometric interpretations;

* incorporates everyday life situations into lesson problems (newspapers, stock market, budgets, sports and so forth);

* teaches geometry and measurement using concrete materials (manipulatives) and calculators;

* requires students to use estimation before trying to solve a problem;

* uses mental arithmetic with the students;

* asks students to justify and explain their answers;

* designs or uses assignments for individual students and groups that require more than a few minutes to complete;

* encourages students to use the available computers in the building when needed;

* helps students to see and experience application of mathematics in other fields through exploration, investigation and discovery;

* encourages students to develop tables, graphs, charts and rules to describe a situation;

* uses activities to help students develop spatial sense and number sense.

Harry Stratigos is program coordinator for Mathematics and Computer Education for the school district of Lancaster, Lancaster, Pennsylvania. He developed the above listing for principals who requested specific teacher behaviors they could look for in implementing the Standards.

Hello From

Richard Kopan, Canadian Representative
Regional Services Committee

Canadian Regional

Congratulations to the Mathematics Council of The Alberta Teachers' Association for hosting the NCTM regional in Calgary in October. Attendance was beyond expectations, and the program was outstanding. A big "thank you" to George Ditto, Lois Marchand and all the hard-working committees.

Calendar

Following is a summary of upcoming activities. If I missed any, or if there is a change, please let me know.

April 17-20	New Orleans	NCTM Annual
May 1-3	Moncton	NBTAMC (New Brunswick)
May 9-11	Toronto	OAME (Ontario)
July 5-7	Toronto	Leadership '91
October 17-19	Newfoundland	MCNTA (Newfoundland)
October 17-19	N.W. (Vancouver)	BCAMT (British Columbia)
October 18	Winnipeg	MAMT (Manitoba)
October 24-26	Regina	STMS (Saskatchewan)
October 31-November 2	Edmonton	MCATA (Alberta)
August 23-25, 1992	Montrêal	NCTM Regional
May 6-8, 1993	Winnipeg	NCTM Regional

NCTM Special Member Products

NCTM now has a 1-800 number for ordering products: 1-800-235-7566

Math Month (April)

NCTM has a packet available to help promote math month. To order it, write to NCTM, 1906 Association Drive, Reston, Virginia 22091.

Professional Teaching Standards

Teaching Standards, a follow-up document to The Curriculum and Evaluation Standards, was released on March 12, 1991. This issue will be free to members who joined by March 1, 1991. The cost to institutional members is $10 until August 1, 1991.

Educational Materials Committee

This committee is still looking for volunteers to preview and review manuscript materials. It you can help, contact Doug Owens, Box 853, Point Roberts, Washington 98281

Until We Meet Again

If you have any questions, concerns or ideas regarding your association and NCTM, feel free to write to me at 23 Lake Crimson Close S.E., Calgary T2J 3K8, or phone me at 271-5240 (res.); 271-8882 (bus.); 299-7049 (FAX).

Membership Application

The Mathematics Council of The Alberta Teachers' Association

A. Name ____________________

Address ____________________

Telephone Number (Home) ____________________

School or Employer ____________________

Grade Level, Specialty ____________________

Local Name and Number ____________________

Teaching Certificate Number ____________________

B. Category of Membership in The Alberta Teachers' Association (check one):

Active ☐ Associate ☐ Student ☐

Life ☐ Honorary ☐

C. Category of Membership in MCATA: New ☐ Renewal ☐

D. Membership Fee Enclosed:
Make cheque payable to The Alberta Teachers' Association (check one)

Regular $25.00 ☐ Affiliate $30.00 ☐

Student $5.00 ☐ Subscription $35.00 ☐

The Concept of Function

The NCTM Standards emphasize the importance of functions, yet research indicates that students have various difficulties with the concept.

Secondary and college students view functions as rules of correspondence given by formulas which allow direct computation of values. Given a particular value for one variable, often denoted by x, students view a function as the rule that allows them to determine the value of another variable denoted by y or f(x). The focus is on function as a symbolic rule, not on function as a particular kind of mathematical relationship. Other representations such as tables, graphs and arrow diagrams are not generally viewed by students as functions. Students do not interpret these other forms as alternative ways to represent the same mathematical relationship that a formula describes.

Piecewise functions, constant functions and functions composed of discrete points present particular problems. Students are strongly "attracted to linearity." When asked to find examples of functions satisfying various constraints, they will provide linear examples almost always, even if it means overriding some of the constraints.

Students do not attend to the subconcepts of function, including domain and range. In one study, students agreed that two functions with identical formulas but different specified domains were the same function. The part of the function definition that requires a unique image for each element of the domain was not at all salient for these students.

Students are comfortable dealing with functions in a "pointwise" manner, but have difficulty when more "global" judgments are required. That is, this research indicates that students can find the value of f(x) for a given value of x, but they have difficulty interpreting the function as a whole scheme. This may be related to the difficulty that calculus students have in looking at secant lines "approaching" the tangent line to a curve at a point, or determining the behavior of a function over an interval.

References

Dreyfus, T., and T. Eisenberg. "Intuitive Functional Concepts: A Baseline Study on Intuitions." Journal for Research in Mathematics Education 13, no. 5 (1982): 360-80.

Markovits, Z., B. Eylon and M. Bruckheimer. "Functions Today and Yesterday." For the Learning of Mathematics 6, no. 2 (1986): 18-24, 28.

Orton, A. "Students' Understanding of Differentiation." Education Studies in Mathematics 15 (1983): 235-50.

Prepared by the Research Advisory Committee of the National Council of Teachers of Mathematics.

MathSoft Announces Exclusive Program for NCTM Members

MathSoft announced that NCTM members and members of NCTM affiliates are eligible for a 10 percent discount on its MathCAD educational software. Normally retailing for $495, the program is discounted for educational buyers to $99-$175 (depending on quantity). In addition to the discounted price, NCTM members will receive a free packet of teaching and laboratory materials that cover prealgebra, algebra, trigonometry, and precalculus problems. The packet is valued at $49. To take advantage of this special offer, call (800) MATHCAD and give the sales representative your NCTM membership number or Affiliated Group identification. This offer expires June 30, 1991.

Dates to Remember

October 31-November 2, 1991

Mathematics Council Annual Conference at the Edmonton Inn

Theme:	"Mathematics: A Meaningful Mosaic"
Keynote Speaker:	Don Fraser, University of Toronto
Fees (including GST):	MCATA Member $74.90
	Non-member $107.00
Co-Chairs:	Bryan A. Quinn, 6 Greenhill Street St. Albert T8N 2B4 Telephone: 460-7763
	Marie Hauk, 315 Dechene Road Edmonton T6M 1W3 Telephone: 487-8841

This is shaping up to be an excellent conference. Efforts should be made to have November 2 declared a P.D. day so that a large number of teachers may attend.

Made-in-Canada Mathematics Assessment

Albertan educators are playing a key role in developing a "made-in-Canada" assessment of student competency in mathematics. The Council of Ministers of Education (Canada) is developing a system of school achievement indicators to help policymakers ensure that education serves the needs of students and society.

One component of the system determines participation rates, graduation rates and similar statistics at provincial and national levels. A second component assesses student achievement in literacy and numeracy.

Alberta and Québec, through their ministries of education, have been given the responsibility of developing suitable criteria by which achievement in literacy and numeracy can be assessed, and of developing suitable assessment instruments for the project.

The assessment framework for numeracy calls for five levels of competency. The fifth level, in particular, denotes a broad range of competence, but there is no implication that this reflects the maximum possible competency in mathematics.

The assessment design for numeracy has the following reporting categories:

* problem solving
* numbers and operations
* algebra and functions
* measurement and geometry
* data management and statistics

Problem-solving items may be associated with one of the major content reporting categories, or they may have minimal specific mathematical content.

As an indication of the size of the numeracy project, we require approximately 2,500 new items to be written. One thousand items will be field-tested in the various provinces, and 250 will make up part of the final assessment forms. We plan to have our initial item pool constructed by September 30, 1991, with field testing (in English and French) in May 1992 and full administration in May 1993.

Mathematics assessments are usually designed to test knowledge of specific course material taught during a semester or a year. The test developers assume that all students taking the test have had an opportunity to learn the material and that the format of the items is familiar to the students.

The School Achievement Indicators Project cannot be completely of this form, as the following restrictions apply:

1. The mathematical content of the open-ended items must be familiar to most 13-year-olds so that level 5 performances are within the capabilities of talented 13-year-olds.

2. There must be scope for a range of valid methods of solution so that success in the item does not depend on the student remembering a specific, narrow formula taught in a particular class.

3. The test must be structured so that below-average student performances are characterized by the successful completion of a limited number of tasks, rather than by unsuccessful attempts at a whole range of tasks.

The test items should reflect the value implicit in the NCTM Curriculum and Evaluation Standards for School Mathematics. For example, we assume that all 13-year-old and 16-year-old students are receiving instruction in the full range of mathematics. Therefore, items asking students to construct mathematical models and to apply these models to real-life situations will be more useful than items that are exercises in pure computation.

Items can be multiple-choice (four or five alternatives, with rationales included for each alternative), numerical-response or open-ended (with attached sample answers and scoring guide). They can be written in English or French.

We will be holding item-writing workshops in the summer, and we also welcome individual items from teachers. Our most pressing needs are for statistics items that concentrate on inference rather than computation, and for high-level problem-solving items that have minimal mathematical content and that permit a range of solutions at different levels of competency.

For further information, contact Jim Brackenbury (Project Director), Phill Campbell (Director) or Jack Edwards, CMEC Numeracy Project, Alberta Education at 427-2948, FAX 422-4200.

The Right Angle

This article is the second in a three-part series discussing the direction of the Mathematics 30 Diploma Examination, with regard to the implementation of the new Mathematics 30 curriculum in September 1991. The first article discussed mathematics as communication, this article discusses mathematics as problem solving, and the third will discuss scoring an open-ended question.

Mathematics as Problem Solving

In keeping with expectations identified in the senior high mathematics curriculum, the examination will reflect mathematics as problem solving. Problem solving is integrated throughout the curriculum, including a set of learner expectations that deals specifically with problem solving and learner expectations within each content strand.

The expectations contained in the program of studies are consistent with the recommendations of the National Council of Teachers of Mathematics:

> In Grades 9-12, the mathematics curriculum should include the refinement and extension of methods of mathematical problem solving so that all students can
>
> * use, with increasing confidence, problem-solving approaches to investigate and understand mathematical content;
>
> * apply integrated mathematical problem-solving strategies to solve problems from within and outside mathematics;
>
> * recognize and formulate problems from situations within and outside mathematics;
>
> * apply the process of mathematical modeling to real-world problem situations.

Focus: In Grades 9-12, the problem-solving strategies learned in earlier grades should have become increasingly internalized and integrated to form a broad basis for the students' approach to doing mathematics, regardless of the topic at hand. From this perspective, problem solving is much more than applying specific techniques to the solution of classes of word problems. It is a process by which the fabric of mathematics, as identified in later standards, is constructed and reinforced. (Curriculum and Evaluation Standards, National Council of Teachers of Mathematics, 1989, p. 137)

The National Council of Teachers of Mathematics describes the evaluation of mathematics as problem solving in the following manner:

The assessment of students' ability to use mathematics in solving problems should provide evidence that they can

* formulate problems,
* apply a variety of strategies to solve problems,
* solve problems,
* verify and interpret results,
* generalize solutions.

(Curriculum and Evaluation Standards, National Council of Teachers of Mathematics, 1989, p. 211)

In the construction of diploma examinations, at present we are limited to using a paper-and-pencil test in assessing problem solving. In 1991, we will be field-testing questions that require students to perform the above tasks.

Example 1:

(a) A student graphed $y = \sin A$. Then she graphed $y = 2 \sin A$. How does the graph of $y = \sin A$ change?

(b) The student then graphed $y = 2 \sin (3A)$. How did the graph of $y = 2 \sin A$ change?

(c) If the parameters a, b, c and d were added to the function $y = \sin A$ so that the function became $y = a \sin [b(A + c)] + d$, how does the graph of $y = \sin A$ change?

This question is designed to lead students from a concrete experience to an abstract experience in mathematics, focusing on the generalization of the effects of parameters a, b, c and d on the function of $y = \sin A$.

If you have any questions or comments, please contact me at 427-2948.

Florence Glanfield
Examination Manager
Mathematics 30

Thoughts on Testing Mathematical Literacy

The Council of Ministers of Education of Canada has requested that an examination be devised to test the mathematical literacy of Canadian students at diverse stages of their education. Before addressing details of the draft document dated October 24, 1990, I wish to make several observations. Some of them are alluded to in the draft document but are not clearly stated.

1. Mathematics is symbolic logic. One makes several assumptions about the (undefined) elements of an abstract set S, the (usually binary) operations upon the elements of S, and the relations between the elements and/or operations. One then makes conjectures that, when proved by applying some sequence of the above-defined operations and relations, are called theorems.

2. Applied Mathematics is mathematics wherein the abstract set and the operations upon its elements are suggested by observations from another area of knowledge which could be another area of mathematics (such as algebra applied to the study of geometry).

3. Mathematical (Theoretical) Science is the validation of the applied mathematics, including the assumptions upon which the mathematical system is based and the evaluation of the relevance of the mathematical theorems proved to the subject that is being examined. Even though the theorems may be valid statements about the system, it is possible that no mathematical step taken in the proof of the mathematical theorem corresponds to any intermediate stage in the evolution of the system being studied.

4. The purpose of the proposed examination is the evaluation of the educational system. Accordingly, one could attempt to correlate the student's innate mathematical ability with the level of his or her acquired mathematical skills. The former might be measured by posing questions (problems) that the student has not been taught to solve. Some of these questions will involve pattern recognition; the most common such questions form the backbone of IQ tests and are similar to those leading to the formulation of the axioms in theoretical science. Other questions could involve proving theorems based on simple sets of axioms; for example, the student could be asked to prove or disprove the associativity of an abstractly defined "multiplication," although one would not pose that question in language the student would have been taught. The acquired skills would be tested by curriculum-based questions. If one is really serious about evaluating the different educational systems, the latter questions should be posed at all levels up to the level that will be challenging to the majority of students in the system in which the curriculum is most comprehensive. One should be interested in the quantity as well as the quality of the acquired mathematical knowledge, given the innate mathematical talent of the participating students.

5. Polya's Method assumes that a problem has been posed, although not necessarily in a neat and compact form. Part of scientific talent is the ability to see questions to ask in a set of facts. The mathematical talent that he

does not mention is the critical evaluation of the problem. It is one thing to show that your solution satisfies the conditions in the posed question, but quite another to show that it is the only solution. One man, upon seeing a sheep in a pasture, observed: "There is a black sheep in that field." His companion observed: "There is a sheep in that field that is black on at least one side." Likewise, the sequence 1, 2, 4, 8, 16, . . . has as its sixth term 31 if it is counting the number of regions into which a circle is partitioned when n points on its circumference are joined in all possible ways, given that no three of these chords meet at a point. A sequence is defined to be a function whose domain is the natural numbers; no finite subset of the sequence can uniquely define this function. Part of "looking back" should be a critical re-evaluation of the assumptions and their consequences. (Apply to the tower problem in the draft document.)

6. Intuition is the (immediate) apprehension of the problem without reasoning. It seems to me that at levels one and two, the only "understanding" of a mathematical problem is intuitive. Of course, that "understanding" may be flawed. Intuition at any level may be flawed, which is the reason that intuition at all levels must eventually be backed up with reason. The intellectual weakness of students at the lower levels of understanding is the inability to justify their ideas, either by invoking logic or experiment.

7. Estimates of solutions to mathematical problems are of two types. The first is the "guesstimate" that one makes to determine the order of magnitude of the numbers involved. Frequently, the exact solution, once obtained, is not very informative. For example, the ratio of the circumference to the radius of a circle is pi, which is neither 3.1415926536 nor 22/7 nor 3 (KINGS 7,23). The latter may have been the first estimate, but 22/7=3.142857 . . . may be the most informative estimate (in its rational form) if one is constructing a circular building. Approximations to the "exact" answer made a posteriori are scientifically useful. The use of both a priori and a posteriori approximations are most likely to be used by people at low levels and high levels of accomplishment. The average person will try to be "exact" but may not succeed.

The above comments have been formulated with a view to modifying the present document rather than proposing a new format. I am not sure that the differences between the five levels of mathematical talent are age related, except that the answers to some of the questions that might be posed to test innate ability are biased by the content of the person's previous education; education improves one's ability to interpret a problem in terms that will facilitate its solution. I also believe that level five is more complex than suggested by the document, and that it is important to try to recognize those students who have a deeper appreciation of the subtleties of mathematics than envisaged there. The most talented will likely make the most significant contributions to society, particularly to its technological development. It may be difficult to pose questions that will distinguish level six from level five using multiple-choice questions, but the style with which written response questions are answered will reflect qualitative differences.

F. A. Baragar

The Student Evaluation Branch, Alberta Education

Initiatives

The Student Evaluation Branch is currently piloting student portfolios as a possible complement, on a sample basis only, to the Achievement Testing Program. These new assessment techniques are being developed to provide a broader picture of what students know and can do. The performance-based assessment activities contained in the portfolio reflect classroom instruction and are believed to be a more developmentally appropriate assessment than the traditional paper-and-pencil test.

There are a number of advantages in using performance-based assessment:

--Permits the assessment of integrated learning across and within subject areas
--Requires active participation of the student
--Integrates classroom instruction and assessment
--Focuses on writing across the curriculum
--Allows for self-assessment
--Provides a more complete picture of what children know and are capable of doing

In addition to portfolio assessment, the Student Evaluation Branch is field-testing a test that reflects classroom instruction. These new developments include

--integrated test items, for example, mathematics concepts couched in a social studies context;

--the use of manipulatives in mathematics and science in particular, perhaps including a "baggy" of easily accessible materials--string, paperclips, plasticine, buttons, ruler, and so on;

--a format reflecting student activity, from filling in circles on multiple-choice answer sheets to shading in graphs or representations of objects.

These initiatives are only in the field-testing stage. Teachers involved in the development of the test and the pilot study have been supportive and enthusiastic.

For more information, phone Dennis Belyk, assistant director, Student Evaluation Branch at 427-0010.

MCATA Executive 1990/91

President
Marie Hauk — Res. 487-8841
315 Dechene Road — Bus. 492-4124
Edmonton T6M 1W3

Past President
Louise Frame — Res. 239-6385
32, 1012 Ranchlands Blvd. NW — Bus. 282-3822
Calgary T3G 1Y1

Vice-President
Bob Hart — Res. 284-3729
16 Rosetree Crescent NW — Bus. 276-5521
Calgary T2K 1M9

Secretary
Dennis Burton — Res. 327-2222
3406 Sylvan Road — Bus. 328-9606
Lethbridge T1K 3J7 — Fax 327-2260

Treasurer
Dick Kopan — Res. 271-5240
23 Lake Crimson Close SE — Bus. 271-8882
Calgary T2J 3K8

delta-K Editors and Publications Directors
John Percevault — Res. 328-1259
2510 22 Avenue S
Lethbridge T1K 1J5

Craig Loewen — Res. 327-8765
Faculty of Education — Bus. 329-2396
University of Lethbridge — FAX 329-2252
4401 University Drive
Lethbridge T1K 3M4

Newsletter Editor
Art Jorgensen — Res. 723-5370
4411 5 Avenue
Edson T7E 1B7

Monograph Editors
Bob Midyette — Res. 282-8916
3343 Boulton Road NW — Bus. 249-3131
Calgary T2L 1M2

Keith Molyneux — Res. 289-8601
2707 48 Avenue NW — Bus. 274-2240
Calgary T2L 1C4

Faculty of Education Representative
Daiyo Sawada — Res. 436-4797
11211 23A Avenue — Bus. 492-0562
Edmonton T6J 5C5

Department of Education Representative
Dick Mitton — Bus. 381-5243
220 5 Avenue S
Bag Service 3014
Lethbridge T1J 4C7

Mathematics Representative
Alvin Baragar — Res. 469-5626
Department of Mathematics — Bus. 492-3398
University of Alberta
Edmonton T6G 2G1

PEC Liaison
Norm Inglis — Res. 239-6350
56 Scenic Road NW — Bus. 285-6969
Calgary T3L 1B9

ATA Staff Adviser
Bill M. Brooks — Res. 256-6314
200, 540 12 Avenue SW — Bus. 265-2672
Calgary T2R 0H4 — or 1-800-332-1280
Fax 266-6190

Membership Director
Diane Congdon — Res. 526-7563
146 Fourth Street SW — Bus. 548-7516
Medicine Hat T1A 4E3

Publicity Director
Bill Davidoff — Res. 627-4283
P.O. Box 574 — Bus. 627-4414
Pincher Creek T0K 1W0

Professional Development Director and NCTM Representative
Wendy Lukawesky — Res. 420-1466
1701, 9808 103 Street
Edmonton T5K 2G4

Directors-at-Large
Bryan Quinn — Res. 460-7733
6 Greenhill Street — Bus. 475-1737
St. Albert T8N 2B4

Alvin Johnston — Res. 723-7247
1026 52 Street — Bus. 723-3992
Edson T7E 1J9

1991 Conference Directors
Marie Hauk
Bryan Quinn

1990 Conference Director
George Ditto — Res. 282-6682
2713 17A Street N.W.
Calgary T2M 3S9

MCATA Elections

Nominations of candidates for the following offices for the 1991-92 school year are now being accepted:

President	Secretary
Vice-President	Treasurer

If you wish to nominate a candidate, please complete the form below and mail it, by May 9, 1991, to Louise Frame, #32, 1012 Ranchlands Boulevard N.W., Calgary, Alberta T3G 1Y1.

If an election is necessary, it will be conducted by mail. Ballots will be sent to all members on or about May 25, 1991.

Ensure an active council by nominating people who will take an active part in making the Mathematics Council a benefit to all mathematics techers.

Nomination Form

We, the undersigned members of the MCATA, nominate ______________________
(name)

of ____
(address)

as a candidate for the office of ________________ in the MCATA for the year 1991-92.

Signature and address of two nominators:

Name ________________ Address ________________

____

Name ________________ Address ________________

____

(Please include a brief resume of the nominee's qualifications for the position on the reverse side of this sheet.)

I accept this nomination ______________________________
(signature of nominee)

Mathematics Educator of the Year

Nomination Form

Nominee's Name ______________________________ Phone ______________________________

Home Address ____

School Address ____

Present Position ____

____

____

____

____

____

Nominated by ______________________________ Phone ______________________________

Address ____

Date ______________________________

This award will be presented at the annual MCATA conference in Edmonton from October 31 to November 2, 1991.

Mail nomination form before September 15, 1991 to

Louise Frame
Chairperson
Award Selection Committee
#32, 1012 Ranchlands Boulevard N.W.
Calgary, Alberta
T3G 1Y1